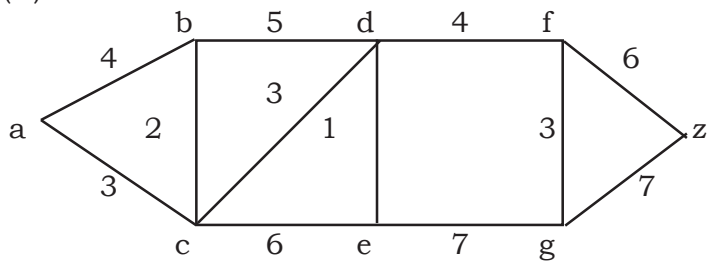
(ii)



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UG Even Semester Examination, September, 2023

COMPUTER SCIENCE

(4th Semester)

Course No.: MCSCC-402

(Discrete Mathematics)

Full Marks: 70

Pass Marks: 28

Time: 3 hours

The figures in the margin indicate full marks for the questions.

Answer any five questions, taking one from each unit.

UNIT-I

1. (a) Without using truth table, show that ¬(PV (¬pΛq) and ¬pΛ¬q, are logicially equivalent. 5

(b) By using truth table show that the following are logically equivalent : 5

(i) pΛ(qVr) and (pΛq) V (pΛr)

(ii) pV (qΛr) and (pVq)Λ(pVr)

(c) Write the negation of the following statements : 4

(i) If it is cold, he wears a hat.

(ii) If it snows, then they do not drive the car

2. (a) Write the converse and inverse of the following statements : 3+3=6

(i) "If the teacher is absent, then some students do not complete their homework."

(ii) "The hometeam wins when even it is raining."

(b) Determine the validity of the following arguments : 4+4=8

(i) If I study, then I will pass.
If I do not go to a movie, then I will study.
I failed.
Therefore, I went to a movie.

(ii) If I like biology, then I will study it.
Either I study biology or I fail the course
If I fail the course, then I do not like biology

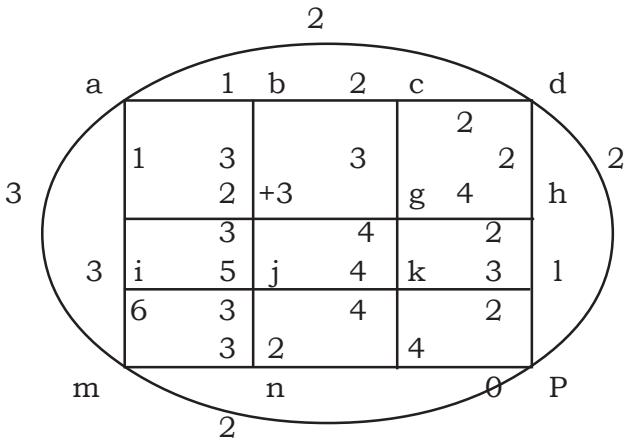
UNIT-II

3. (a) Define subgroup. If H and K are subgroups of a group G, show that $H \cap K$ is a subgroup of G. 5

(b) Let G be the set of all 2x2 matrics, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where a, b, c, d are real numbers such that $ad-bc \neq 0$. Show that G is a group with respect to matrix multiplication.

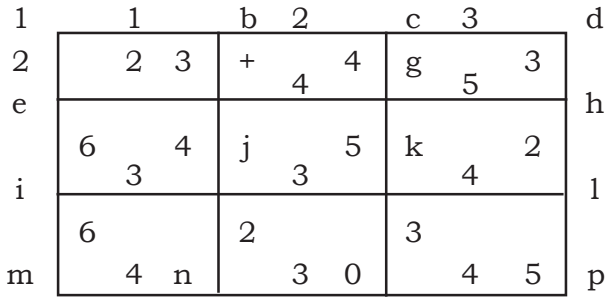
(c) Let S= N X N. Let * be the operation on S defined by $(a,b) * (a', b') = (aa', bb')$. 4

(i) Show that S is a semigroup w.r.t * .



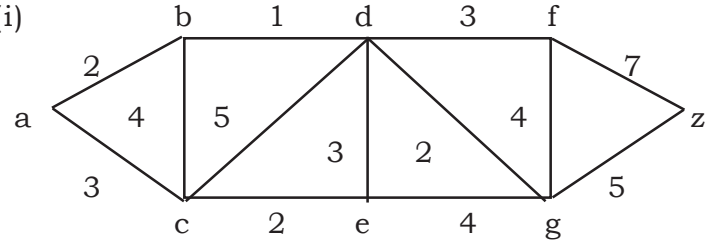
5+4=9

(ii)



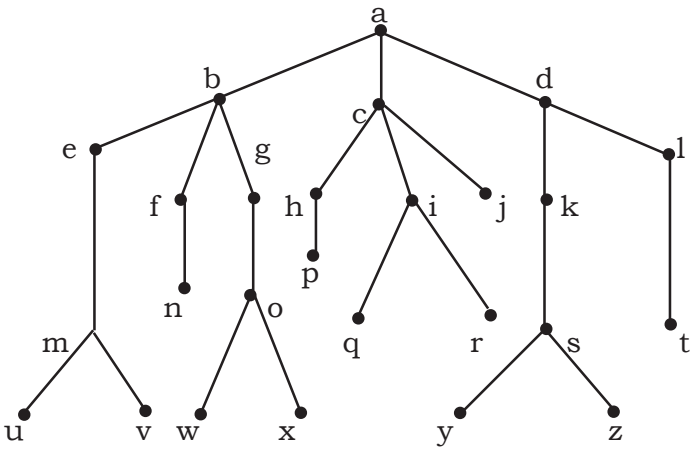
10. Write Dijkstra's algorithm for finding the shortest path of a weighted graph. Using Dijkstra's algorithms, find the shortest path between the vertices 'a' to 'z' for the following graphs: 7+7=14

(i)



(c) Define complete and regular graphs with example. 3

8. Write the pre-order, in-order and post-order traversal algorithm's for a rooted tree. Find the order in which the vertices of the tree will be visited by pre-order traversal, in-order traversal and post-order traversal : 14



UNIT-V

9. (a) Define spanning tree and minimum spanning tree. Write the Prim's algorithm for finding minimum spanning tree of a weighted graph. 5

(b) By Kruskal algorithm, find a minimum spanning tree for the following graphs : 5+4=5

(ii) Define $f : (S, *) \rightarrow (Q, X)$ by $f(a,b) = a/b$. Show that f is a homomorphism.

(b) Let G be the group of all 2 x 2 real matrics $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with $ad - bc \neq 0$, under matrix multiplication. Let 5

$H = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \mid ad \neq 0 \right\}$

Then show that H is a subgroup of G.

(b) Define Abelian group. Give an example of a group which is not Abelian group with justrification. 4

(c) If G is a group, then prove the following -

(i) The identity element of G its unique

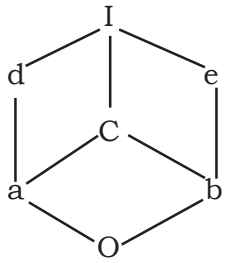
(ii) Every $a \in G$ has a unique inverse in G.

(iii) For all $a, b \in G$, $(a.b)^{-1} = b^{-1}.a^{-1}$

UNIT-III

5. (a) Define distributive lattice. Let L be a bounded distributive lattice. Then show that complements are unique if they exist. 5

(b) Consider the lattice L in figure given below: 5



(i) Which monzero elements are join irrducible?

(ii) Which elements are atoms?

(iii) Which of the following are sublattices of L?

L_1 = {0, a, b, I}
L_2 = {0, a, e, I}
L_3 = [a, c, d, I}
L_4 = {0, c, d, I}

(iv) Is L distributive?

(c) Define linearly ordered set. State whether each of the following subsets of N are linearly ordered : 4

(i) {24, 2, 6}
(ii) N= {1, 2, 3, 4, ...}
(iii) {2, 8, 32, 4}
(iv) {2, 4, 8, 16, 20}

6. (a) Define Boolean algebra. Show that the following are equivalent in a Boolean algebra: 5

(i) a+b= b (ii) a*b = a

(iii) a′+b= 1 (iv) a*b′ = 0.

(b) Define sum of products from and complete sum of products from in Boolean algebra. 4

(c) Express each of the following Boolean expression as sum of products and then in its complete sum of products form : 5

(i) E(x, y, z) = x(xy′ + x′y + y′z)

(ii) E(x, y, z) = Z(x′ + y) + y′

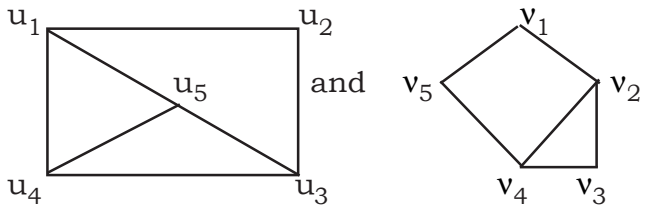
UNIT-IV

7. (a) Define incidence matrix and adjancency matrix. Draw the undirected graphs represented by the following adjacency matrix: 6

(i) $\begin{bmatrix} 0 & 2 & 3 & 0 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix}$

(b) Define isomorphic graphs. Check whether the following pairs of graphs are isomorphic or not: 5

(i)



(ii)