UNIT-V

9. (a) Define confidence coefficient. Let (X_1, X_2, ..., X_n) be a random sample from U(0, θ). Obtain a (1-α) level confidence interval of θ based on the nth order statistics. 2+5=7

(b) Explain the concept of the shortest length confidence inverval. Let (X_1, X_2, ... X_n) be a random sample from U(0, θ). Obtain the shortest length confidence interval of θ. 2+5=7

10. (a) Explain the advantage of interval estimation over point estimation. Deduce the (1–α) level confidence interval for θ when (x_1, x_2, ..., x_n) be a random sample from the distribution. 2+5=7

$$f(x;\theta) = e^{-(x-\theta)};\ x>0,\ -\infty < \theta < \infty$$

(b) Determine the (1–α) level of confidence interval of σ^2 when (X_1, X_2, ..., X_n) be a random sample from a normal distribution n(μ, σ^2) when (i) μ is known and (ii) μ is unknown. 3.5+3.5=7

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PG Even Semester Examination, September, 2023

STATISTICS

(2nd Semester)

Course No: STA-551
(Statistical Inference-I)

FM: 70
Pass Marks : 28
Time: 3 Hours

The figures in the right margin indicate full marks for the question.
Answer five questions, taking one from each unit.

UNIT-I

1. (a) Differentiate between estimator, estimation and estimate with an example. 4

(b) If T is an unbiased estimator for parameter θ, show that T^2 is a biased estimator for θ^2. 4

(c) Let (X_1, X_2, ..., X_n) be a random sample from Bernoulli (θ). Find the unbiased estimator of θ^2 and θ (1–θ). 6

2. (a) If T is a consistent estimator of the parameter θ, then show that T^2 is also a consistent estimator of θ^2. 6

(b) Define sufficient statistic and minimal sufficient statistic. Explain Lehman and Scheffe to obtain minmal sifficient statistic. 2+2+4=8

UNIT-II

3. (a) State and prove Rao-cramer's inequality. 8

   (b) Find the Rao-cramer's lower bound for the variance of an unbiased estimator of θ in the probability density function. 6

   $$f(x;\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}} ; x > 0, \theta > 0$$

4. (a) State and prove Rao-Blackwell theorem. 8

   (b) Using Rao-Blackwell theorem find an unbiased estimtor of $e^{-\theta}$ for the poisson (θ) distribution. 6

UNIT-III

5. (a) Describe the method of moments for estimating parameters of a distribution. Discuss when the estimates obtained by the method of moments ar one identical with those of maximum likelihood estimates. 7

   (b) Let $(X_1, X_2, \ldots, X_n)$ be randem sample from a distribution with density function. 7

   $$f(x;\theta) = \theta(\theta+1)\ x^{\theta-1}\ (1-x);\ 0<x<1<\theta>0$$

   Determine the estimate of θ by the method of moments.

6. (a) Show that maximum likelihood estimators are functions of the sufficient statistics if a sufficient statistcs exist. 4

   (b) Let $(X_1, X_2, \ldots, X_n)$ be n sample observations from an exponential distribution with mean $^1/_\theta$. Find the maximum likelihood estimator of the (i) median of this distribution and that of (ii) $P(X>10)$. 3+3=6

   (c) Examine a situation when maximum likelihood method fails and explain how how you tackle such situations. Do this with an example. 4

UNIT-IV

7. (a) Define the concept of critical region. Let X has a pdf of the form

   $$f(x) = \begin{cases} \theta x^{\theta-1};\ 0 < x < \theta \\ 0\ ,\ \text{otherwise} \end{cases}$$

   Find the power function of the test to test the simple null hypothesis $H_0 : \theta=1$ against the alternative simple hypothesis $H_1: \theta=2$ using a random sample X_1 and X_2 of size n=2 and defining the critical region to be

   $$w = (x_1, x_2) : \frac{3}{4x_1} < x_2 \Big\}$$

   (b) State and prove the Neyman-Pearson Lemma. 7

8. (a) Explain with example (i) Best Critical Region, (ii) Uniformly Most Powerful Test. 3.5+3.5=7

   (b) Obtain the best critical region for testing $H_0 : \mu = \mu_0$ against the alternative simple hypothesis $H_1 : \mu - \mu_1 : \mu = \mu_1$, for the normal distribution with unit variance. Hence find the power of the test. 5+2=7

Turn Over