UNIT-V

9. (a) If X_1, X_2, ..., X_n be a random sample of size ‘n’ from U(0, θ) obtain a (1–α) level confidence interval of θ based on the n^{th} order statistics. 6

(b) Define confidence interval. Let X_1, X_2, ..., X_n be a random sample from N (μ, σ^2). Obtain (1–α) level confidence interval for m when.

(i) σ^2 is known.

(ii) σ^2 is unknown. 2+3+3

10. (a) Define shortest length confidence interval with an example. 5

(b) Let X_1, X_2, ..., X_n be a random sample from N(μ, σ^2). Suppose μ is known, obtain the shortest length 1–α level confidence interval of σ^2. 9

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PG Even Semester Examination, September, 2023

STATISTICS

(2nd Semester)

Course No: STSCC-204
(Statistical Inference)

FM: 70
Pass Marks : 28
Time: 3 Hours

The figures in the right margin indicate full marks for the question.
Answer five questions, taking one from each unit.

UNIT-I

1. (a) What is unbiasedness? If X_1, X_2, ..., X_n is a random sample from a Bernoulli variate X taking value 1 with probability θ and O with probability 1–θ. Show that, $\frac{T(T-1)}{n(n-1)}$ is unbiased estimatator of θ^2, where

$T= X_1 + X_2 + ... + X_n$. 2+4

(b) Define the concept of consistency of estimators. Let X_1, X_2, ..., X_n be a ranom from a uniform distribution.

$U(0,\theta)$ as $\theta \in (\Theta)$. Let

$M_n = \max(X_1, X_2, ..., X_n)$

Show that $M_n \xrightarrow{b} \theta$

Turn Over

If $Y_n = 2\overline{X}$, show that Y_n is also consistent for θ. 2+6

2. (a) Do consistency & unbiasedness of estimators necessarily go together? Illustrate your answer with suitable examples. 7

(b) What do you mean by efficiency of estimators?

If T is the most efficient estimator of θ and T' be any other estimator with efficiency e, then show that

$\mathrm{Var}(T-T') = (\frac{1}{e} - 1)\ \mathrm{Var}(T)$ 2+5=7

UNIT-II

3. (a) State and prove the Rao-Blackwell theorem. 7

(b) Prove that if 'U' be a non-empty class of unbiased estimator of θ with $E(T^2) < \infty$ for $\theta \in \Theta$, there exists atmost one UMVUE of θ. 7

4. (a) State and prove the Cammer Rao inequality. 7

(b) State & prove the Lehmann-Scheffe theorem. 7

UNIT-III

5. (a) What is maximum likelihood estimator? State 3 properties of the maximum likelihood estimator. 1+6

(b) Obtain the maximum likelihood estimator for β in random samples from the gamma distribution.

$$f(x)= \frac{1}{\beta^{\alpha}\overline{|\alpha}}\ e^{-x/\beta}\ x^{\alpha-1}\ ,\ 0 < x < \alpha$$

With known. Also find the variance of the estimate. 7

6. (a) Let $X_1, X_2, \ldots, Xn$ be iid $U(0,\theta)$, $\theta > 0$. Find the MLE of θ and check if the estimate unbiased and consistant. 7

(b) Explain the method of moments. Let $X_1, X_2, \ldots, X_n$ be the iid Bin (n,p) random variables where both n and p are unknown. Obtain their estimators by using the method of moments. 7

UNIT-IV

7. (a) Define : 6

(i) Critical region

(ii) Type-I & Type-II errors

(iii) Level of significance

(b) State & prove the Neyman Pearson lemma. 8

8. (a) Define (i) Most Powerful test, (ii) Uniformly Most Powerful test. 3+3=6

(b) For the p.df $f(x;\theta) = \theta e^{-\theta x}$, $\theta>0$, $x>0$ derive the critical region for testing the null hypothesis for testing $H_0 : \theta = 1.5$ against the alternative hypthesis $H_1 : \theta>1.5$. 8