PG Odd Semester (CBCS) Exam., December—2017

PHYSICS

(1st Semester)

Course No. : PHYCC–102

(Mathematical Physics—I)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Prove the Cauchy-Schwarz inequality. 6

(b) *(i)* Show that the eigenvalues of a unitary operator are 1. 2

(ii) Show that the eigenvectors of a unitary operator corresponding to different eigenvalues are orthogonal. 3

(c) Show that under similarity transformation $A \quad SAS^{1}$, the set of eigenvalues of the similar operators remains unaltered. 3

2. *(a)* Show that the following sets of vectors are linearly independent :

$\vec{A} \quad \hat{i} \quad \hat{j}, \vec{B} \quad 2\hat{i} \quad 4\hat{j}, \vec{C} \quad \hat{i} \quad 2\hat{j} \quad 3\hat{k}$

Apply Schmidt's orthogonalization procedure to obtain an orthonormal basis. 3+5=8

(b) Diagonalize the following matrix : 6

$$A \quad \begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}$$

UNIT—II

3. *(a)* Determine the analytic function

$f(z) \quad u(x, y) \quad iv(x, y)$

where $u(x, y) \quad 4xy \quad 3x \quad 2$. 4

(b) Prove Cauchy's integral formula. Also obtain the expression of the nth order derivative of an analytic function about a point in the complex plane. 7+3=10

4. *(a)* Expand in Laurent series the function

$f(z) \quad e^{3z} (z \quad 1)^{3}$

about its singularity. 6

(b) State Cauchy's residue theorem. Evaluate the following definite integral using residue theorem : 2+6=8

$$\int_0 \frac{dx}{1 \quad x^4}$$

UNIT—III

5. *(a)* Expand $f(x) \quad x \quad x^2$ for $\quad x \quad$ in a Fourier series and hence show that

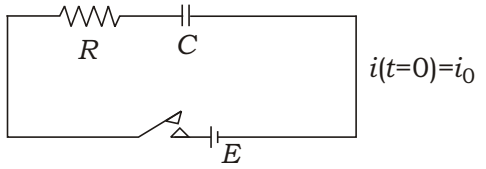
$$(2) \quad \frac{2}{6}$$

where $(k) \quad \sum_{n \quad 1} \frac{1}{n^k}$ is the Riemann zeta function. 6+4=10

(b) Obtain the Fourier transform $F(k)$ of the Gaussian distribution function $f(x) \quad Ne^{\quad x^2}$, where N and are constants. 4

6. *(a)* Obtain the Fourier expansion of the function $f(x) \quad |x|$ in the interval $2 \quad x \quad 2$. 7

(b) Using Laplace transform, find the decay of current $i(t)$ in the following circuit : 7



UNIT—IV

7. *(a)* Find the roots of $x^4 \quad x \quad 10 \quad 0$ using Newton-Raphson method, correct to three decimal places. 6

(b) Find an approximate value of $y(0\ 2)$ using fourth-order Runge-Kutta method, given that $\frac{dy}{dx} \quad x \quad y^2$ and $y \quad 1$ when $x \quad 0$. 8

8. *(a)* Compute the value of the definite integral $\int_{0\ 2}^{1\ 4} (\sin x \quad \ln x \quad e^x)\, dx$ using Simpson's rule. 7

(b) Illustrate Picard's method of successive approximation by solving the differential equation $\frac{dy}{dx} \quad x \quad y$ with the initial conditions $x_0 \quad 0$, $y_0 \quad 1$. 7

UNIT—V

9. *(a)* Show that the cylindrical coordinate system is orthogonal. 4

(b) Express velocity $\vec{v}$ and acceleration $\vec{a}$ of a particle in cylindrical coordinates. 3+3=6

(c) Derive the transformation law for Christoffel symbol of the first kind. 4

10. *(a)* Obtain the expression for ^{2} in orthogonal curvilinear coordinates and hence express ^{2} in spherical coordinates. 8+3=11

(b) Show that any tensor can be expressed as a sum of a symmetric and a skew-symmetric tensor. 3

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