PG Odd Semester (CBCS) Exam., December—2017

PHYSICS

(1st Semester)

Course No. : PHYCC–103

(Quantum Mechanics—I)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* What is Compton effect? Obtain the expression of Compton shift of wavelength. 2+5=7

(b) Find the wavelength of an electron whose kinetic energy is twice its rest energy. 3

(c) Using Heisenberg's uncertainty principle, estimate the groundstate energy of the hydrogen atom. 4

2. *(a)* State and prove Ehrenfest's theorem. 2+5=7

(b) Show that $m\frac{d}{dt} \quad x \quad\quad p$, where the symbols have their usual meanings. 4

(c) State the postulates of quantum mechanics. 3

UNIT—II

3. Examine the bound-state motion of a particle of mass, m and energy, E in the potential

$$V(x) \quad \begin{matrix} O, & a \quad x \quad a \\ V_0, & x \quad a, x \quad\quad a \end{matrix}$$

where $(V_0 \quad E \quad O)$, by solving the Schrödinger equation in the above regions. Discuss the result briefly. 14

4. Explain briefly the hydrogen atom by solving its Schrödinger equation. 14

UNIT—III

5. *(a)* State and prove generalized uncertainty principle. 6

(b) Find the harmonic oscillator state vector corresponding to the minimum uncertainty product. 5

(c) Show that $(\quad L_x)(\quad L_y) \quad \frac{\hbar}{2}| \quad L_z \quad |$, where the symbols have their usual meanings. 3

6. *(a)* For a quantum mechanical operator A, prove that

$$\frac{d}{dt}\langle A\rangle \quad \left\langle \frac{A}{t}\right\rangle \quad \frac{1}{i\hbar}\langle [A, H]\rangle$$ 5

(b) Compare the Schrödinger, Heisenberg and Dirac interaction picture. Show that the time evolution for operators ($_H$) in the Heisenberg picture is given by

$$\frac{d \ _H}{dt} \quad \frac{\ _H}{t} \quad \frac{1}{i\hbar}[\ _H, H]$$ 3+6=9

UNIT—IV

7. *(a)* Explain briefly the Hilbert space and its properties in quantum mechanics. 3

(b) Show that the Hermitian operators always have a real eigenvalue and the eigenfunctions of a hermitian operator that correspond to different eigenvalues are orthogonal. 4

(c) Prove that the Hamiltonian of a particle is Hermitian. 3

(d) Write a short note on 'matrix representation of operators'. 4

8. Using the creation (a) and annihilation (a) operators, show that the nth state vector of the harmonic oscillator is given by

$$_n \quad \frac{1}{\sqrt{n!}} \quad \frac{m}{\hbar}^{\frac{1}{4}} (a \ )^n \exp \quad \frac{m}{2\hbar} x^2$$

with the groundstate

$$_0(x) \quad \frac{m}{\hbar}^{\frac{1}{4}} \exp \quad \frac{m}{2\hbar} x^2$$

where the symbols have their usual meanings. 14

UNIT—V

9. *(a)* Write short notes on the following : 2+2=4

(i) Reflection symmetry

(ii) Time-reversal symmetry

(b) Show that

$$_{lm} \quad \frac{1}{\sqrt{2}} e^{im} \ (\ 1)^m \sqrt{\frac{(2l \ \ 1)(l \ \ |m|)!}{2(l \ \ |m|)!}} P_l^{|m|} (\cos \ )$$

are simultaneous eigenfunctions of L^2 and L_z belonging to eigenvalues $l(l \ \ 1)\hbar$ and $m\hbar$ respectively. 10

10. *(a)* For a spinor $(j \quad \frac{1}{2})$, obtain the J_x, J_y, J_z and J^2 matrices and hence show that $\vec{J} \quad \frac{1}{2}\hbar \vec{\ }$, where J represents the total angular momentum and represents the Pauli spin matrices. 7

(b) Find the Clebsch-Gordon coefficients for the system with the two angular momentum values $\frac{1}{2}$ and $\frac{1}{2}$. 7

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