PG Odd Semester (CBCS) Exam., December—2017

PHYSICS

(3rd Semester)

Course No. : PHYCC–301

(Mathematical Physics—II)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Show that $\frac{d^2y}{dx^2}$ y 0 with $y(0)$ 1 and $y(\ )$ 5 has no solution. 3

(b) State the basic existence and uniqueness theorem for the differential equation $\frac{dy}{dx}$ $f(x,y)$ with $y(x_0)$ y_0. Check the theorem for $\frac{dy}{dx}$ $\frac{y}{\sqrt{x}}$, $y(0)$ 2. 3+3

(c) Calculate the current in a circuit given by differential equation $L\frac{di}{dt}$ Ri E with $i(t$ 0) 0. 5

2. *(a)* Solve

$$x\frac{d^2y}{dx^2} \quad 2x\frac{dy}{dx} \quad 2y \quad x\ln x$$ 9

(b) Solve

$m\frac{d^2y}{dt^2}$ ky mg with $y(0)$ y_0 and $\dot{y}(0)$ 0 5

UNIT—II

3. *(a)* For the system of equations

$$\frac{dx}{dt} \quad \sin y \text{ and } \frac{dy}{dt} \quad \cos x$$

find the fixed point and the equation of trajectory. 4

(b) Write the normal forms for the following bifurcations :

(i) Saddle node

(ii) Transcritical

(iii) Pitchfork

Discuss the stability of fixed points for each case. 6

(c) In Lorentz system, find the value of r when trajectory settles down similar to that of strange attractor. 4

4. (a) In linear approximation of a system given by $\dot{x}$ $f(x, y)$ and $\dot{y}$ $g(x, y)$, check the condition under which the fixed point will become either a node or a saddle point. 8

(b) Define limit cycle and its stability. 3

(c) State Poincaré-Bendixson theorem and apply it on $\dot{x}$ x $4y$, $\dot{y}$ x y^3 to check the existence of closed orbit. 3

UNIT—III

5. (a) Define Hermite polynomial. Check that $(e^{2xt}$ $x^2)$ is the generating function of it. 3

(b) Show that the following recurrence relations are satisfied for Hermite polynomial :

(i) $H_n(x)$ $2nH_{n\ 1}(x)$

(ii) $H_{n\ 1}(x)$ $2xH_n(x)$ $2nH_{n\ 1}(x)$ 6

(c) Show that Hermite polynomial satisfies the orthogonality relation

$$e^{\ x^2} H_m(x)H_n(x)dx \quad \sqrt{\ }2^n n!\ _{mn}$$ 5

6. (a) Define Green's function. Calculate the Green's function associated with $\frac{d^2x}{dt^2}$ w^2x $f(t)$, where x satisfies the boundary condition $x(0)$ 0 and $x(a)$ 0. 6

(b) Solve the Poisson's equation

$$^2\ (\bar{r})\quad 4\ (\bar{r})$$

using Green's function, where $(\bar{r})$ is charge density at point $\bar{r}$, $(\bar{r})$ is electrostatic potential. 8

UNIT—IV

7. (a) Define non-Abelian group. Show that the minimum order of a non-Abelian group is 6. 4

(b) State and prove Schur's lemma. 4

(c) Verify that the number of non-equivalent irreducible representation of a group is equal to number of classes in the group. 6

8. (a) Prove that the order of a finite group must be an integer multiple of the order of any of its sub-groups. 4

(b) Show that every representation of a finite group is equivalent to a unitary representation. 2

(c) Prove that the sum of squares of the dimensions of all inequivalent irreducible representations of a finite group is equal to the order of the group. 6

(d) Define generator of SO(2). 2

UNIT—V

9. *(a)* Show that

$$\frac{P(B|A)}{P(C|A)} \quad \frac{P(B)\,P(A|B)}{P(C)\,P(A|C)}$$

where P(.) is absolute probability of one event P(.1.) is conditional probability. 3

(b) If 1000 people each select a number at random between 1 and 500, what is the probability that 3 people select number 25? 3

(c) Prove that the most probable number of successes of n independent trials in binomial distribution is $[N_p \quad P]$, where P is probability of success in each particular trial and [ ] denotes the integral part. 6

(d) Define the principle of least squares of errors. 2

10. *(a)* State and prove Laplace-De Moivre limit theorem. 9

(b) State Poisson's law and find the probability that the ace of spade will be drawn from the deck of cards at least once in 104 consecutive trials. 5

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