PG Odd Semester (CBCS) Exam., December—2018

PHYSICS

(1st Semester)

Course No. : PHYCC–102

(Mathematical Physics—I)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Define the inner product of two vectors and hence show that the inner product is anti-linear w.r.t. pre-factor. 2+3=5

(b) State and prove Schwartz inequality. 4

(c) Given the vectors in $\mathbb{R}^3$

$$_1 \begin{matrix} 1 \\ 1 \\ 2 \end{matrix}, \; _2 \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}, \; _3 \begin{matrix} 0 \\ 1 \\ 1 \end{matrix}$$

Out of these, construct an orthonormal set of vectors by using Gram-Schmidt orthogonalization. 5

2. *(a)* Suppose U and H are two matrices related by

$$U \quad e^{iH}$$

Show that H is Hermitian if U is unitary. 2

(b) Show that the Eigen value of a Hermitian matrix are real and the Eigen vectors of a matrix belonging to different Eigen values are orthogonal. 3+3=6

(c) Find the Eigen values and Eigen vectors of the matrix

$$\begin{matrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{matrix}$$

6

UNIT—II

3. *(a)* Obtain the Cauchy-Riemann condition. Determine whether the function $f(z) \quad e^z$ satisfies the Cauchy-Riemann condition. 3+2=5

(b) Prove that an analytic function has derivative of all orders. 6

(c) Evaluate

$$\oint_C \frac{\sin z}{z - \frac{}{4}}^{3} dz$$

where C is a unit circle. 3

4. *(a)* Obtain the series expansion of

$$f(z) \quad \frac{1}{2 \quad z}$$

for *(i)* $|z| \quad 2$, *(ii)* $|z| \quad 2$. 3+3=6

(b) Find the residue of the function

$$f(z) \quad \frac{1}{e^{z} \quad i}$$ 3

(c) Evaluate the integral

$$\int_0 \frac{d}{5 \quad 4\cos^2}$$ 5

UNIT—III

5. Let $f(x)$ be a function of period 2 such that

$$f(x) \quad \begin{matrix} 0, & x & 0 \\ x, & 0 & x \end{matrix}$$

(a) Sketch a graph of $f(x)$ in the interval $3 \quad x \quad 3$.

(b) Find the Fourier series expansion.

(c) By giving appropriate values to x, show that

$$\frac{}{4} \quad 1 \quad \frac{1}{3} \quad \frac{1}{5} \quad \frac{1}{7}$$ 3+6+5=14

6. *(a)* What are Fourier and Laplace transforms? 2+2=4

(b) Find the Fourier transform of the exponential decay functions

$f(t) \quad 0$ for $t \quad 0$ and

$f(t) \quad Ae^{\quad t}$ for $t \quad 0$, ($\quad 0$) 5

(c) Find the Laplace transform of the function $f(t) \quad e^{at}$. 5

UNIT—IV

7. *(a)* Obtain Newton-Raphson formula and explain Newton-Raphson method for finding root of algebraic equations with examples. Discuss the limitations of this method. 2+5+1=8

(b) Explain the concept of numerical integration using Simpson's $\frac{1}{3}$ rule and hence evaluate the integral

$$\int_1 e^x \, dx \qquad 6$$

8. *(a)* Explain briefly the Runge-Kutta method for solving non-linear differential equation. Using this method, estimate $y(0\ 4)$ while it is given that

$$\frac{dy}{dx} \quad x^2 \quad y^2 \text{ with } y(0) \quad 0$$

(take $h \quad 0\ 2$). 10

(b) Write a FORTRAN program to find the average of the first N Fibonacci numbers. 4

UNIT—V

9. *(a)* A co-ordinate system (u_1, u_2, u_3) is defined by $u_1 \quad \sqrt{x^2 \quad y^2}$, $u_2 \quad \tan^{\ 1} \frac{y}{x}$, $u_3 \quad z$, where (x, y, z) are Cartesian co-ordinates.

(i) Find the scale factors.

(ii) Show that the co-ordinate system is orthogonal.

(iii) Find the expression for , $\vec{A}$ and $\vec{A}$. 10

(b) Verify Stokes theorem for the vector $\vec{A} \quad y\hat{i} \quad x\hat{j} \quad z\hat{k}$ taking the hemisphere of radius 1 with centre at the origin above the XY-plane. 4

10. *(a)* Define contravariant, covariant and mixed tensors. 2+2+2=6

(b) What is metric tensor? Define the covariant derivative of a tensor. 2+2=4

(c) Write a detailed note on the various fundamental operations with tensors. 4

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