2018/ODD/08/21/PHY–301/222

PG Odd Semester (CBCS) Exam., December—2018

PHYSICS

(3rd Semester)

Course No. : PHYCC–301

(Mathematical Physics—II)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* What are the singular, regular-singular and essential singular points of a linear homogeneous second-order differential equation? Find the singular points and mention the type of the singular point of the following differential equations : 3+3=6

(i) $(1 \quad x^2)y \quad 2xy \quad l(l \quad 1)y \quad 0$

(ii) $x^2 y \quad xy \quad (x^2 \quad a^2)y \quad 0$

(b) State and explain the Picard's existence and uniqueness theorem. Find the solution of the differential equation using Picard's iteration method : 3+5=8

$$y \quad 2y\,; \quad y(0) \quad 1$$

2. *(a)* Explain the Frobenius method for series solution. Solve the equation $y \quad 2y \quad 0$ using power series method. 3+5=8

(b) Solve the following system of first-order equations : 6

$$x_1 \quad x_1 \quad 2x_2\,; \quad x_1(0) \quad 0$$

$$x_2 \quad 3x_1 \quad 2x_2\,; \quad x_2(0) \quad 4$$

UNIT—II

3. What are the non-linear dynamical systems? What is fixed point of a dynamical system? Discuss the following terms with appropriate diagrams : 2+12=14

(a) Stable and asymptotically stable

(b) Node and saddle

(c) Focus and centre

4. *(a)* Explain briefly the Poincare-Bendixson theory. What is the implication of this theory? 3+1=4

(b) What is bifurcation? Mention a few types of bifurcation. 2+2=4

(c) Write down the Lorentz equation. Why is it so important? What are its basic properties? 6

UNIT—III

5. (a) Discuss the Green's function method for solving inhomogeneous linear equation. Consider the following boundary value problem and solve it by Green's function method : 3+5=8

$$y \quad x^2 \; ; \; y(0) \quad 0 \text{ and } y(1) \quad 0$$

(b) Write down the hypergeometric equation. Define the hypergeometric function. How is the series solution of the hypergeometric equation written in terms of hypergeometric function.? 2+2+2=6

6. (a) What are the Legendre function of first kind, $P_n(x)$ and second kind, $Q_n(x)$? What is the generating function of Legendre polynomial, $P_n(x)$? Using generating function of $P_n(x)$, show that

$$P_n(\quad x) \quad (\quad 1)^n P_n(x)$$ 4+1+3=8

(b) What is the series form of the Laguerre polynomials, $L_n(x)$. Find the expressions for $L_0(x)$, $L_1(x)$, $L_2(x)$ and $L_3(x)$. What is the orthogonality property of $L_n(x)$? Evaluate

$$_0 \quad e^{\quad x} L_n(x)\, dx$$ 1+2+1+2=6

UNIT—IV

7. (a) Check whether the following sets form groups : 2+2=4

(i) Set of all $n \quad n$ unitary matrices under matrix multiplication

(ii) Set of all transformations which leave a physical system invariant

(b) Prove that—

(i) if H is a proper subgroup of G and H consists of complete classes, then it is an invariant subgroup;

(ii) an element of a group G constitutes a class by itself, if it commutes with all the elements of G;

(iii) there can be two non-isomorphic groups of order four. 4+3+3=10

8. *(a)* Consider the symmetry group of transformations of an equilateral triangle.

(i) Write down the symmetry operations and workout the group multiplication table.

(ii) Identify all the subgroups and the invariant subgroup. 5+4=9

(b) What is Lie group? Show that the transformation as

$$x \quad ax \quad b, \quad a \quad 0$$

from a Lie group. Find the generators. 5

UNIT—V

9. *(a)* A function $f(x)$ is defined as

$$f(x) \quad \begin{array}{lll} 0 & ; & x \quad 2 \\ \frac{1}{18}(2x \quad 3) & ; & 2 \quad x \quad 4 \\ 0 & ; & x \quad 4 \end{array}$$

Show that it is a probable distribution function. 2

(b) If m and $_r$ denote the mean and rth moment of a Poisson distribution, then prove that

$$_{r\ 1} \quad rm \ _{r\ 1} \quad m\frac{d \ _r}{dm}$$ 4

(c) A random variable x has a standard normal distribution . Prove that

$$\Pr(|x| \quad k) \quad 2[1 \quad (k)]$$ 3

(d) Assuming the errors in a series of observations to have a Gaussian distribution, obtain expression for the probable error of a single observation and that in the arithmatic mean of observations. 5

10. *(a)* Show that the co-variance of two independent variables is always zero but the reverse is not true. 4

(b) Show that the distribution for which the characteristic function is $e^{\ |t|}$ has the density function

$$f(x) \quad \frac{1}{\ } \frac{dx}{1 \quad x^2}, \qquad x$$ 3

(c) If X and Y are two independent random variables following the Poisson distribution with parameters $_1$ and $_2$ respectively, then show that $X \quad Y$ is also a Poisson variable with parameter $_1 \quad _2$. 3

(d) If X and Y are two independent unit normal variates, find the probability density function of the following : 4

(i) $\dfrac{X \quad Y}{\sqrt{2}}$

(ii) $\dfrac{X \quad Y}{\sqrt{2}}$

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