Or

8. (a) Solve $xy - \frac{dy}{dx} = y^3 e^{-x^2}$ 3

(b) Solve $\frac{dy}{dx} - \frac{tany}{1+x} = (1+x)\, e^x\, secy$ 4

(c) Find the general solution and singular solution of the differential equation $y = px + \frac{a}{p}$, $p = \frac{dy}{dx}$ 4

(d) Solve $Sin\left(x\frac{dy}{dx}\right) cosy = cos\left(x\frac{dy}{dx}\right) siny + \frac{dy}{dx}$ 4

9. (a) Solve $\frac{d^2y}{dx^2} + a^2y = cos\, ax$ 5

(b) Solve $(D^3 + 4D^2 + 4D)\, y = 8e^{-2x}$, $D \equiv \frac{d}{dx}$ 4

(c) Solve $(D^4 - 8D)\, y = x^2 + e^{2x}$, $\frac{d}{dx} \equiv D$ 5

Or

10. Solve the following differential equation :

(a) $\frac{d^2y}{dx^2} + a^2y = sec\, ax$ 5

(b) $x^2 \frac{d^2y}{dx^2} + 5x \frac{dy}{dx} + 3y = e^{-x}$ 5

(c) $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = 6x^5$ 4

★★★

UG (CBCS) EVEN SEMESTER EXAMINATION, 2024

EDUCATION
(B.Sc. B.Ed.)
4th Semester

Course No. : BSMP – 402
(Vectors, Statics and Dynamics)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) Find the value of 'a' so that $\lim_{x\to 1} f(x)$ exists, where

$$f(x) = \begin{cases} 3x+5 & , \ x \le 1 \\ 2x+a & , \ x > 1 \end{cases}$$

(b) Examine the continuity of 3

$$f(x) = |x - b| \text{ at } x = b$$

(c) Evaluate $\lim_{x\to 0} \frac{1-cosx}{2x^2}$ 3

(d) Evaluate $\lim_{x\to 2} \frac{\sqrt{3-x}-1}{2-x}$ 3

(e) Define uniform continuity of a function. Prove that if a function f satisfy Lipschitz condition, then the function is uniformly continuous. 2

Or

2. (a) State Rolle's theorem verify Rolle's theorem for the function $f(x) = 2x^2 + 1$, $x \in [-2, 2]$ 1+2=3

(b) State Lagrange's mean value theorem. Verify Lagrange's mean value theorem for the function $f(x) = 2x^2 - 4x + 5$, $x \in [0, 2]$ 1+2=3

(c) Examine the validity and conclusion of Lagrange's mean value theorem for the function 3

$$f(x) = x(x-1)(x-2), \quad x \in \left[0, \frac{1}{2}\right]$$

(d) Find $\lim_{x\to\infty} x\, e^{-x}$ 3

(e) Find $\lim_{x\to\infty} x^{\frac{1}{x}}$ 2

3. (a) Expand e^x in an infinite series in powers of x. 4

(b) Expand $\cos x$ in an infinite series in power of x. 5

(c) Find the Maclaurin's series for $\sinh x$. 5

Or

4. (a) Prove that for the parabola $y^2 = 4ax$, the subnormal is constant and the sub-tangent varies as x. 5

(b) For the cardioid $r = a(1 - \cos\theta)$, prove that polar subtangent $= 2a \sin^2 \frac{\theta}{2} \tan \frac{\theta}{2}$ 4

(c) Find the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ 5

5. (a) Find the reduction formula for $\int \sin^n x\, dx$, where n is a positive integer. Evaluate $\int_0^{\pi/2} \sin^5 x\, dx$ 2+3=5

(b) Find the reduction formula for 5

$$\int_0^{\pi/2} \sin^m x . \cos^n x\, dx$$

(c) Evaluate $\int_0^{\pi/2} \sin^7 x \cos^4 x\, dx$ 4

Or

6. (a) Find the surface area of the paraboloid of revolution of the parabola $y^2 = 4ax$, about x-axis and bounded by the section $x = a$. 5

(b) What is the entire area enclosed by the curve $y = x^3$ and the x-axis, between $x = 0$ and $x = 1$. 3

(c) Find the area enclosed by the curve $y = x^2 - 5x + 4$ and the x-axis, between $x = 1$ and $x = 5$. 3

(d) What is the entire area enclosed by the curve $r = a\cos 2\theta$. 3

7. (a) Find the differential equation of the function $y = (A + Bx)\, e^{kx}$; A, B are arbitary constant. 3

(b) Determine order, degree and linearity of the differential equation $\left(\frac{d^2x}{dt^2}\right)^2 + \frac{d^2x}{dt^2} + t\frac{dx}{dt} = 0$ 3

(c) Solve $ydx + (x + \cos y)\, dy = 0$ 4

(d) Solve $x\cos x \frac{dy}{dx} + y(x\sin x + \cos x) = 1$ 4