respectively, show that where R is the radius of the earth. 7

9. (a) A particle describes the equiangular spiral $r = ae^{\theta}$ in such a manner that the radial acceleration is zero. Prove that the speed and the magnitude of acceleration are each proportional to r. 7

(b) A particle falls under gravity in a medium in which the resistance is proportional to velocity. If the particle, falls vertically downwards from a position of rest, find the velocity and displacement in time t. 7

Or

10. (a) A particle is projected vertically upwards with velocity v, and the resistance of the air produces a retardation kv^2, where v is the velocity. Show that the velocity v_1 with which the particle will return to the point of projection is given by $\frac{1}{{v_1}^2} = \frac{1}{v^2} + \frac{1}{w^2}$, where w is the terminal velocity. 7

(b) The force acting on a particle moving along x-axis is given by the equation $F=1+2x+3x^2+4x^3+5x^4$. Calculate the work done when x changes from 0 to 5. Force is in Newtons and x in metres. 7

★★★

UG (CBCS) EVEN SEMESTER EXAMINATION, 2024

EDUCATION
(B.Sc. B.Ed.)

6th Semester

Course No. : BSMP – 603
(Vector Calculus, Statics and Dynamics)

Full Marks : 70
Pass Marks : 28

Time : 2 hours

The figures in the margin indicate full marks for the questions

1. (a) Find the value of the constant d, such that vectors $-3\hat{k}$ and $3\hat{i}+d\hat{j}+5\hat{k}$ are coplanar. 3

(b) Show that $(\bar{\alpha}+\bar{\beta}).\{(\bar{\beta}+\bar{\gamma})\times(\bar{\gamma}+\bar{\alpha})\} = 2[\bar{\alpha}\,\bar{\beta}\,\bar{\gamma}]$ 3

(c) Prove that 3

(d) Prove that for the curve 3
$\bar{r} = 2a\cos t\,\hat{i} + 2a\sin t\,\hat{j} + bt^2\,\hat{k}, [\,\bar{r}\;\bar{r}\;\bar{r}\,] = 8a^2bt$

(e) If , then find 2

Or

2. (a) Find the directional derivative of $\varphi = xy^2z + 4x^2z$ at (-1, 1, 2) in the direction $2\hat{i}+\hat{j}-2\hat{k}$ 3

(b) If the vectors and be irrotational, then show that the vectors is solenoidal. 3

(Turn Over)

(c) Show that the vector $\bar{F} = 3y^4z^2\,\hat{i} + 4x^3z^2\,\hat{j} - 3x^2y^2\,\hat{k}$ is solenoidal. 3

(d) Show that curl grad , where $f = x^2y + 2xy + z^2$ 3

(e) Show that the vector is both solenoidal and irrotational. 2

3. (a) Three forces P, Q, R act along the sides of a triangle formed by the lines $x + y = 3$, $2x + y = 1$ and $x - y = -1$. Find the equation of the line of action of the resultant. 7

(b) The algebraic sums of the moments of a system of coplanar forces about points whose co-ordinates are (1, 0), (0, 2) and (2, 3) referred to rectangular axes are H, 2H and 3H respectively. Find the tangent of the angle which the direction of the resultant force makes with the axis of x. 7

Or

4. (a) A small bead p can slide on a smooth elliptic wire; it is attracted towards the foci S and H by forces proportional to $(SP)^m$ and $(HP)^n$ respectively, find the positions of equilibrium. 7

(b) Three equal uniform rods each of weight w, are smoothly pin-jointed to form an equilateral triangle ABC which hangs in equilibrium from A. Find the actions of the rods on the pins. 7

5. (a) Find the C.G of the area of a cardioid, when the density varies as the cube of the distance from the origin. 7

$f y \hat{i} = \bar{0}\ x\ \hat{j}$

(b) Show that the locus of the C.G of all equal segments cut off from a parabola is an equal parabola. 7

Or

6. (a) Find the differential equation of the path of the particle moving under a central force. 7

(b) If the orbit described by a particle under a central force to the origin by $r^n \cos n\theta = a^n$, find the law of force. 7

7. (a) Find the tangential and normal components of the acceleration of a particle describing a plane curve. 7

(b) A particle describes a path with an acceleration μ/y^3 which is always parallel to the axis of y and directed towards the x-axis. If the particle e projected from a point (0, a) with the velocity $\sqrt{\mu}/a$ parallel to x-axis, show that the path described is a circle. 7

Or

8. (a) A particle of mass m moves on a straight line under an attraction 4mx towards a point 0 on the line, where x is the distance from 0. If the particle starts from rest at $x = 5$, show that when it first reaches $x = 3$, the magnitude of the velocity is 8. 7

(b) A particle is projected vertically upwards with a velocity v from the earth's surface. If h and H be the greatest heights attained by the particle moving under uniform and variable accelerations