UG (NEP) EVEN SEMESTER EXAMINATION, 2024

COMPUTER SCIENCE

2nd Semester

Course No. : CS-DSM - 151

(Mathematics-II)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer five questions selecting one from each unit)

Unit - I

$\lim_{x \to 0} \frac{sinx - tanx}{x^3}$

1. (a) If f : A $\to$ R and if 'c' is a cluster point of A, then show that '*f*' can have only one limit at 'c'. 5

   (b) Define limit of a function at any point c. Using defition of limit, show that : 5

   $$\lim_{x \to 2} \frac{x^3 - 4}{x^2 + 1} = \frac{4}{5}$$

   (c) Evaluate, $\lim_{x \to \infty} \left\{x - \sqrt{(x-a)(x-b)}\right\}$, where a, b are real numbers. 4

2. (a) Using definition of limit show that $\underset{x \to c}{\text{Lt}}\, x^2 = c^2$ 5

   (b) Evaluate the following, limit's : 3+3+3=9

   (i)

(ii) $\lim_{x\to 0} \frac{1-cos^3x}{tan^2x}$

(iii) $\lim_{x\to 0} \frac{cosecx - cotx}{x}$

Unit - II

3. (a) Define continuous function. Using ∈-δ definition, show that the function $f(x) = x$, is a continuous function. 5

(b) Discuss the continuity of the following function's at the point indicated : 3+3+3=9

(i) $f(x) = \begin{cases} x^2+2x+1, & if \quad 0 \le x < 1 \\ 4, & if \quad x = 1 \\ 2x^3-3x^2+5, & if \quad x > 1 \end{cases}$

at the point $x = 1$

(ii) $f(x) = \begin{cases} x^2+2x+3, & if \quad 0 \le x < 2 \\ 15, & if \quad x = 2 \\ x^2-2x-3, & if \quad x > 2 \end{cases}$

at $x = 2$

(iii) $f(x) = \begin{cases} x, & if \quad 0 < x < 1 \\ 2-x, & if \quad 1 \le x \le 2 \\ x-\frac{1}{2}x^2, & if \quad x > 2 \end{cases}$

at $x = 2$

4. (a) If $y = \cos(m \sin^{-1} x)$, show that 6

(i) $(1- x^2)\, y_2 - xy_1 + m^2y = 0$

(ii) $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} + (m^2 - n^2)\, y = 0$

(c) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent series. 4

10. (a) Examine whether the following series : 6

$$2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \cdots + \frac{(n+1)x^n}{n^3} + \cdots$$

is convergent or divergent if $x > 0$.

(b) Define Riemann sum and Riemann integration of a function. 4

(c) If f(x) is a Riemann integrable function, on the closed interval [a, b], then show that the value of the integral is uniquely determined. 4

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(b) Find the maximum and minimum values of the following expression : 2+3+3=8

(i) $x^3 - 9x^2 + 24x - 12$

(ii) $2x^3 - 21x^2 + 36x - 20$

(iii) $1 + 2\sin x + 3\cos^2 x$

Unit - III

5. (a) Define derivative of a function at any point c. If f : I $\rightarrow$ R, has a derivative at c$\in$I, then show that 'f' is continuous at 'c'. 2+4=6

(b) Using e d definition of derivative, prove the following identities : 3+3+2=8

(i) $\frac{d}{dx}(\tan x) = sec^2 x$

(ii) $\frac{d}{dx}\left(\phi(x).\psi(x)\right) = \phi(x)\,\psi'(x) + \psi(x).\phi'(x)$

(iii) $\frac{d}{dx}(\cos x) = -sinx$

6. (a) If $f(x) = \begin{cases} x, & if \quad 0 < x < 1 \\ 2 - x, & if \quad 1 \le x < 2 \\ x - \frac{1}{2}x^2, & if \quad x \ge 2 \end{cases}$

show that $f(x)$ is continous at $x = 1$, but it is not differentiable at $x = 1$. 7

(b) Show that the function $f(x)$, defined by 4

$$f(x) = \begin{cases} 3 + 2x, & -3/2 \le x < 0 \\ 3 - 2x, & 0 \le x \le 3/2 \end{cases}$$

is not differentiable at $x = 0$

(c) If $f(a) = 2, f'(a) = 1, g(a) = -1$, and $g'(a) = 2$, then find the value of $\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$ 3

Unit - IV

7. (a) Define convergent sequence. Show that the sequence $\left(\frac{1}{n}\right)$ converges to 0. $1\frac{1}{2} + 3\frac{1}{2} = 5$

(b) Show that every convergent sequence of real number's is bounded. 5

(c) Define cauchy sequence and monotone sequence. 4

8. (a) Show that every convergent sequence of real number's is a cauchy sequence. 5

(b) Let (x_n) and (y_n) be sequences of real numbers, that converges to x and y respectively. Show that the sequence (x_n+y_n), (x_n-y_n) and $(x_n.y_n)$ converges to $x+y$, $x-y$ and $x.y$ respectively. $2\frac{1}{2} + 2\frac{1}{2} + 4 = 9$

Unit - V

9. (a) Write limit comparison test, ratio test and root test. 6

(b) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is a convergent series. 4