PG EVEN SEMESTER EXAMINATION, 2024

STATISTICS

2nd Semester

Course No. : STS - 552

(Linear Models and Regression)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer five questions taking one from each unit)

Unit - I

1. (a) Consider a simple linear regression model $y_i = \alpha + \beta x_i + u_i$, derive an expression for standard error in case of individual prediction of y for a given value of x. Also explain how the confidence statements can be made about individual y for a given x. 9

   (b) Define a simple linear regression model. What are the assumptions of this model? 5

2. (a) Show that least square estimators are best linear unbiased estimators (BLUE). 7

   (b) How will you check the assumption of regression model using residual plots? Explain. 7

Unit - II

3. (a) Define multiple linear regression model. Write down the assumptions of this model. 7

   (b) Define k-variable linear regression model. Write down the assumptions of this model. 7

4. Define multivariate normal distribution.

   If y ~ Nn (Xβ, σ^2 In) where X is (n x k) matrix of rank k then

   i) $\hat{\beta} \sim N_k [\beta, \sigma^2 (X'X)^{-1}]$

   ii)

   iii) is independent of s^2

   iv)

   $\frac{(n-k)}{\sigma^2} s^2 \sim \chi^2_{(n-k)}$

   2+3+2+3+4=14

Unit - III

5. (a) Write a note on ridge regression in estimating the parameters of a regression model. 6

   (b) Define autocorrelation. What are the causes of autocorrelation? 8

6. (a) Consider

   $y_i = \beta_1 + \beta_2 x_i + u_i$, $E(u_i, u_j) \neq 0$ be a simple two-variable regression model, where β_1 and β_2 are the parameters and u_i has the first order auto-regressive scheme. Show that

   i) $E(u_i) = 0$

   ii) $v(u_i) = \frac{\sigma_i^2}{1-\rho^2}$

   iii) $\text{cov}(u_i, u_{i+s}) = \rho^2 \frac{\sigma_i^2}{1-\rho^2}$; $s \neq 0$ 9

   (b) Discuss the consequences of multicollinearity. 5

Unit - IV

7. Define Growth models and derive the models for logistic and Gompertz models. 4+5x2=14

8. If the model $Y = \frac{\theta_1}{\theta_1 - \theta_2} [e^{-\theta_2 t} - e^{-\theta_1 t} + \epsilon]$

   Estimate the coefficients using Tayler Series expansion. 14

Unit - V

9. Define odd, odds ratio and log odds ratio and its interpretation by giving an example. 4+5+5=14

10. (a) Define different types of logistic regression models. 6

    (b) Derive the mean and variance by using he link function for binomial and poisson models. 8

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