(b) State and prove Wald's equation. 5

(c) Explain when a stochastic process is called a modified, delayed or general renewal process. 5

10. (a) For large t, show that the average number of renawals per unit converges to $1/\mu$. 5

(b) Show that the renawal function M satisfies the equation 5

$$M(t) = F(t) = \int_0^t M(t-x)\, d\, F(x)$$

(c) Show that the distribution of N(t) is given by

$$P_n(t) = P_n\,[\,N(t) = n\,] = F_n(t) - F_{n+1}(t)$$

and the expected number of renawals by 4

★★★

PG EVEN SEMESTER EXAMINATION, 2024

STATISTICS

2nd Semester

Course No. : STS - 553

(Stochastic Processes)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer five questions taking one from each unit)

Unit - I

$M(t) = \sum_{n=1}^{\infty} F_n(t)$

$P = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}$

1. (a) Define Markov chain. 2

(b) Classify the states of Markov chain. 2+2+2=6

(c) Show that if state K is persistant null then, for every chain $p_{jk}^{(n)} \to 0$ as $n \to \infty$. If state k is a periodic non-null, then $p_{jk} \to F_{jk}/\mu_{kk}$ 6

2. (a) Consider a Markov chain with state space S = (0,1,2) and transition probability matrix

Classify the states and chain. 6

(b) Write a note on higher transition probability. 5

(c) Find the relationship between f_{jk} and p_{jk}; symbols have their usual meaning. 3

Unit - II

3. (a) Define counting process. Write the properties of counting process. 2+2=4

(b) Show that the time between occurance of two events is exponential with parameter λ. 5

(c) Write a note on Birth-Death process. 5

4. (a) Prove the continuation property of a Poisson process. 5

(b) Write a note on Yule-Furry process. 4

(c) Define martingale. When a process is said to be a martingale. 2+3=5

Unit - III

5. (a) Explain one-dimensional random walk with an example. 7

(b) Discuss Brownian motion and its applications. 7

6. (a) Show that one-dimensional random walk is recurrent for $p = q = {}^1/_2$ and transient for $p \neq q$, where p and q have their usual meanings. 7

(b) Describe Gambler's ruin problem. Derive expression for the probability of Gambler's ruin. 7

Unit - IV

7. (a) When a stochastic process is called a branching process? Give example. 3+2=5

(b) For the branching process, $P[X_n = K] = bc^{k-1}$; $k=1, 2, \ldots$; find the distribution of the number of offsprings of the *n*-th generation. 6

(c) Define chance of extinction. 3

8. (a) In a branching process, the number of offsprings per individual has a binomial distribution with parameters (2, p). Starting with a single individual, calculate

i) the extinction probability

ii) the probability that the population becomes extinct for the first time in 3rd generation. 5

(b) Prove that if $m \leq 1$; the probability of ultimate extinction is 1. If $m > 1$; the probability of ultimate extinction is the +ve root less than unity of the equation $S = P(s)$. 9

Unit - V

9. (a) Define renewal process. In renewal process, when we get Poisson distribution as a particular case. 4