8. (a) For the following variance-covariance matrix deduce the first principal components 7

$$\Sigma = \sigma^2 \begin{bmatrix} 1 & \rho & \rho & \cdots & \rho \\ \rho & 1 & \rho & \cdots & \rho \\ \vdots & \vdots & \vdots & & \vdots \\ \rho & \rho & \rho & \cdots & 1 \end{bmatrix}; 0 \leq \rho < 1$$

(b) Explain in details the multivariate regression model and also deduce the maximum likelihood estimate for the coefficient matrix B. 7

Unit - V

9. (a) Derive the appropriate test for testing the null hypothesis $H_0 : \Sigma_{ij} = 0 \quad \forall\ i \neq j$ (i.e. independence of several sub vectors) using the likelihood ratio. criterion. 7

(b) Explain the different methods of cluster formation. 7

10. (a) What is MANOVA? State the assumptions of MANOVA. 2+5=7

(b) Explain the concept & terminology of two-way MANOVA. 7

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PG EVEN SEMESTER EXAMINATION, 2024

STATISTICS

4th Semester

Course No. : STS - 652

(Multivariate Analysis)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer five questions taking one from each unit)

Unit - I

1. (a) Find the maximum likelihood estimators of the mean vector and the variance-covariance matrix of a multivariate normal distribution. 7

(b) Let $X = (X_1, X_2, \ldots X_p)'$ be a vector of random variates, define

$Y_1 = X_1$, $Y_i = X_i - X_{i-1}$, $i = 3, 2, \ldots p$

If Y_i's are mutually independent, each with multiple variance σ^2 prove that,

$$\text{tr}\,(\Sigma) = \sigma^2 \frac{p(p+1)}{2}$$

where, Σ is the variance-covariance matrix of X. 7

2. (a) Let X is a random p-vector with mean vector μ and dispersion matrix Σ. Prove that X is distributed as $N_p(\mu, \Sigma)$ if for every coefficient l, $l'x$ is distributed as $N_1(l'\mu, l'\Sigma l)$

Also prove that if X is so distributed, $X'\Sigma^{-1}X$ is distributed as non central χ^2 with p degrees of freedom of non-centrality paramter $\mu'\Sigma^{-1}\mu$. 8

(b) Suppose , similarly

$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$ and

with usual meanings as in multivariate analysis. Then show that, if the distribution of X_1 given $X_2 = x_2$ is normal having mean

$\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(x_2 - \mu_2)$ and the covariance matrix is 6

Unit - II

3. (a) Define the Wishart distribution. State and proof the additive property of the Wishart distribution. 2+5=7

(b) Explain the impact of linear transformation in case of Wishart distribution. 4

(c) Explain the formation of a trinomial distribution. 3

4. (a) Deduce the marginal distribution of any x_j when $(x_1, x_2, \ldots x_k)$ follows multinomial distribution. 7

(b) Obtain the variance-covariance matrix of the multinomial distribution. 7

Unit - III

5. Starting from the definition of Cannonical Correlation deduce the i[th] set of Cannonical variates between and . 14

6. (a) What is discriminant analysis? Deduce an expression for the discriminating function of two populations setup. 2+5=7

(b) What is latent variable? State the assumptions of a factor analysis model. 2+5=7

Unit - IV

7. (a) Show that Mahalonobis D^2 is invariant under non-singular linear transformation. 3

(b) Deduce the distributional form of the Hotelling's T^2. 6

(c) State and prove the multivariate central limit theorem. 5