PG EVEN SEMESTER EXAMINATION, 2024

STATISTICS

2nd Semester

Course No. : STS - 551

(Statistical Inference–I)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer five questions taking one from each unit)

Unit - I

1. (a) Explain the difference between estimate and estimator of a parameter. Define unbiased estimator of a parameter. Provide an example to show that unbiased estimator need not be unique. 2+2+3=7

   (b) State and prove the invariance property of constant estimators using invariance property, show that $\frac{x}{n}\left(1-\frac{x}{n}\right)$ is consistant for p(1 - p) where p is the probability of success in Bernoulli population. 5+2=7

2. (a) Let $(x_1, x_2, \ldots, x_n)$ be independent and identical

random sample with $E(x_i) = \mu$ and $E(x_i^2) < \infty$. Examine where

$$T(x_1, x_2, \ldots, x_n = \frac{2}{n(n+1)} \sum_{j=1}^{n} j\, xi$$

is consistant for μ. 7

(b) What do you mean by sufficient statistic. Explain with an example.

Let $(x_1, x_2, \ldots, x_n)$ be a random sample from a population with pdf. 3+4=7

$f(x ; \theta) = \theta x^{\theta-1}$; $0<x<1$, $\theta>0$ show that

$T = \prod_{j=1}^{n} x_i$ is sufficient for θ.

Unit - II

3. (a) State and prove Rao-cramor's inequality and explain its importance in statistical inference. 10

(b) Obtain the minimum variance bound estimator for a in normal population N (μ, σ^2), where σ^2 is known. 4

4. (a) State and prove Rao-Blackwell theorem and discuss its uses in statistical inference. 7

Unit - V

9. (a) Describe pivotal quantity method of constructing confidence interval. 7

(b) Show that for the exponential distribution $f(x, \theta) = \theta e^{-\theta x}$; $x>0$, $\theta>0$, central confidence limits for θ for large samples of size n and 95% confidence coefficient are 7

$$\left(1 \pm \frac{1.96}{\sqrt{n}}\right).\ \frac{1}{x}$$

10. (a) Develop a general method of obtaining confidence intervals. Obtain a $100(1-\alpha)\%$ confidence interval for large sample size for the parameter v of the poisson distribution. 5

(b) Discuss the construction of shortest expected length confidence interval. 5

(c) Explain how interval estimation differs from point estimation. 4

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(b) Let $(x_1, x_2, \ldots, x_n)$ be a random sample of size n drawn from the distribution. 7

$f(x; \theta) = \theta^x (1 - \theta)^{1-x}$; $x = 0,1$; $0<\theta<1$. Find uniformly minimum variance unbiased estimator of θ, θ^2 and $\theta(1 - \theta)$

Unit - III

5. (a) If a sufficient estimator exists, show that it is a function of the maximum likelihood estimator. 5

(b) Obtain maximum likelihood estimate of θ in $f(x, \theta) = (1 + \theta)\, x^{\theta}$; $0<x<1$, $\theta>0$ based on an independent sample of size n. Examine whether this estimate is sufficient for θ 5

(c) Estimate α and β using method of moment for the distribution

$$f(x; \alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)}\, x^{\alpha-1}\, e^{\beta x} \; ; 0 \le x < \infty$$ 4

6. (a) Describe the method of moments and discuss when the estimates obtained by the method of moments are identical with those of maximum likelihood estimates. 5

(b) Describe the maximum likelihood method of estimation and state at least three of its optimal properties. 5

(c) Examine a situation when maximum likelihood method fails and explain how you tackle such situations. 4

Unit - IV

7. (a) State and prove Nayman-pearson Lemma for testing a simple null hypothesis against a simple alternative hypothesis. 10

(b) Explain the following terms

(i) Type I and Type II errors

(ii) power of a test

(iii) simple and composite hypothesis.

(iv) level of significance 4

8. (a) Use the Neyman-pearson Lemma to obtain the region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta_1 > \theta_1$ in case of a normal population N (θ, σ^2), when σ^2 is known. 7

(b) Suppose you are testing the null hypothesis $H_0;\theta=1$ against $H_1;\theta=2$ by means of a single observed value of x for the distribution

$f(x; \theta) = \frac{1}{\theta}$; $0 \le x < \theta$. What would be the sizes of the type I and type II errors, if you choose the interval (i) $x \ge 0.5$ and (ii) $1<x<1.5$ as the critical regions of Also obtain the power function of the test. 7