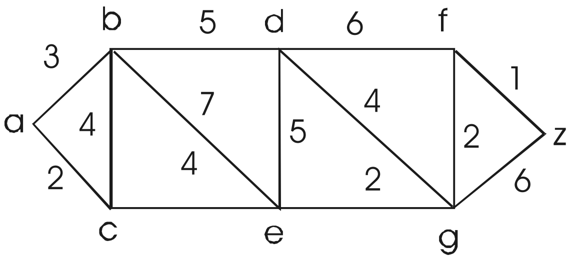
(ii)



*****

UG/Integrated Even Semester Examination, 2025

COMPUTER SCIENCE

(4th Semester)

Course No.: CS-DSC-253/MCSC-402

(Discrete Mathematics)

Full Marks: 70

Pass Marks: 28

Time: 3 hours

The figures in the right margin indicate full marks for the question. Answer any five questions selecting one from each unit.

UNIT-I

1. (a) Define converse, contrapositive and inverse of a conditional statement. Write the contrapositive, the converse and the inverse of the following statement : 6

“The home team wins whenever it is raining”.

(b) Without using truth table, show that $\neg(P \vee (\neg P \wedge q)$ and $\neg P \wedge \neg q$ are logically equivalent. 4

(c) Write the negation of the following statements : 4

(i) “At lest 10 inches of rain fall today in Delhi”.

(ii) If it shows, then they donot drive car.

2. (a) Using truth table show that the following propositions are logically equivalent: 3+2+2+7

(i) $PV\ (q \wedge r)\ \equiv\ (PVq) \wedge (PVr)$

(ii) $\neg(PVq)\ \equiv\ \neg P \neg q$

(iii) $\neg(P \wedge q)\ \equiv\ \neg PV \neg q$

(b) Determine the validity of the following arguments : 3½+3½=7

(i) If I study, then I will pass. If I donot go to a movie, then I will study.

I faillled.

(ii) If I like Biology, then I will study it. Either I study Biology or I will fail the course.

If I fail the course, then I donot like Biology.

UNIT-II

3. (a) If G is a group, then prove the following:

(i) The identity element of G is unique.

(ii) Every $a \in G$, has a unique inverse in G.

(iii) For every $a \in G$, $(a^{-1})^{-1} = a$.

(iv) For all a, $b \in G$ $(a.b)^{-1} = b^{-1}.a^{-1}$. 7

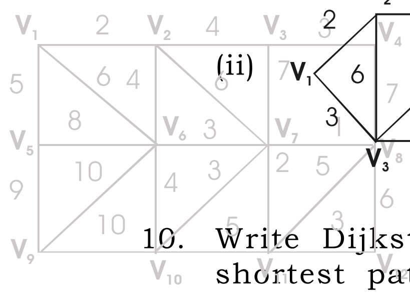
(b) Define group. Let G be the set of all 2x2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where a, b, c, d are real numbers such that $ad - bc \neq 0$. Show that G is a group with respect to matrix multiplication. 7

UNIT-V

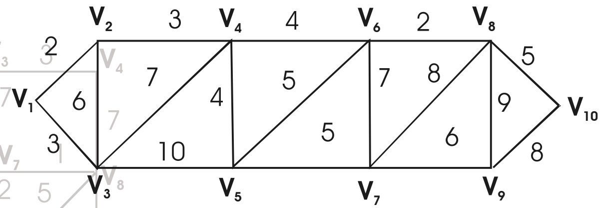
9. (a) Define spanning tree and minimum spanning tree. Write the prime's algorithm for finding minumum spanning tree of a weighted graph. 5

(b) Usind Kruskal algorithm, find a munimum spanning tree for the following graphs. 5+4=9

(i)

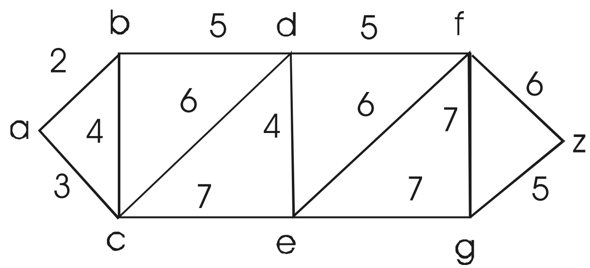


(ii)



10. Write Dijkstra's algorithm for finding the shortest path of a weighted graph. Using Dijkstra's algorithm's, find the shortest path between the vertices 'a' to 'z' for the following graphs. 4+5+5=14

(i)



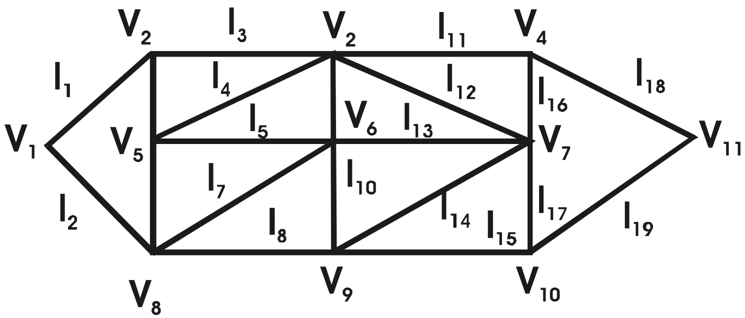
(i) $E(x,y,z) = x(xy+y^1+x^1y)$

(ii) $E(x,y,z) = (x^1+y^1)+x^1y$

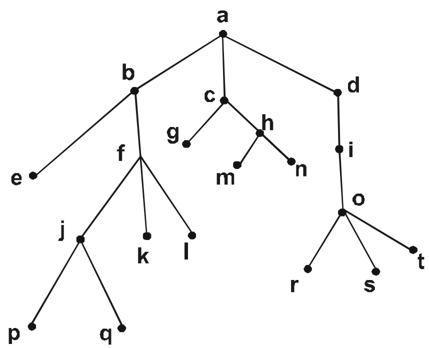
UNIT-IV

7. (a) Define Path, cycle, Bipartite graph, connected graph and regular graph with example. 7

(b) Define incidence matrix and adjacency matrix. Represent the graph given below by both incidence matrix and adjacency matrix. 7



8. Write the pre-order, in-order and post-order traversal algorithm's for a rooted tree. Find the order in which the vertices of the tree will be visited by pre-order traversal, in-order traversal and post-order traversal. 14



4. (a) Show that the following four matrices form a group under matrix multiplication : 5

$$A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad B=\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix} \quad C=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

$$D=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$

(b) Show that the set $S=\{0,1,2,3,4,5\}$, forms a finite abelian group, under addition module 6. 5

(c) Define ring and zero division. 4

UNIT-III

5. (a) Define these diagram of a partially ordered set. Draw the Harse diagram of D_{36}. 4

(b) Define consistent enumeration. Show that these exist a consistent enumeration for any finite poset A. 5

(c) Define distributive lattice. Let 'L' be a bounded distributive lattice. Then show that complments are unique if they exist. 5

6. (a) Prove that the following are equivalent in a Boolean Algebra. 6

(i) $a+b=b$ (ii) $a*b=a$

(iii) $a^1+b=1$ (iv) $a*b^1=0$

(b) Define sum of products form and complete sum of products form in Boolean algebra. 4

(c) Express each of the following Boolean expression a sum of products and its complete sum of products form. 4