UG/Integrated Even Semester Examination, 2025

COMPUTER SCIENCE

(2nd Semester)

Course No.: CS-DSM-151

(Mathematics-II)

Full Marks: 70

Pass Marks: 28

Time: 3 hours

The figures in the right margin indicate full marks for the question. Answer any five questions selecting one from each unit.

UNIT-I

1. (a) Evaluate the following limits : 2½+2½=5

(i) $\lim\limits_{x\to\propto} (x - x2 + x)$

(ii) $\lim\limits_{x\to\frac{\pi}{4}} (\text{seczx} - \text{tanzx})$

(b) Using definition of limit, show that

$\mathop{Lt}\limits_{x\to 2} (2x-2)=4$ 5

(c) Define cluster point. Find all the cluster point's of the interval (3,5). 4

2. (a) If $\mathop{Lt}\limits_{x\to a} f(x)= l$ and $\mathop{Lt}\limits_{x\to a} g(x)= l'$, wherel and l' are finite real number's, then show that

(i) $\mathop{Lt}\limits_{x\to a} (f(x) + g(x)) = l+l'$

(ii) $\underset{x\to a}{\text{Lt}}\ (f(x) - g(x)) = l - l'$ 3+3=6

(b) Deine limit of a function at any point C. Using definition of limit show that

$\underset{x\to c}{\text{Lt}}\ x^2 = C^2$ 5

(c) If f:A→R and if 'C' is a cluster point of A, then show that 'f' can have only one limit at 'C'. 4

UNIT-II

3. (a) Define continuous function. Let A R,and f : A→R, g : A→R, such that 'f' and 'g'are continuous at any point C∈A, then show that the function f+g and f–g are also continuous at 'C'. 2+2+2=6

(b) Discuss the continuity of the following function's at the point indicated : 5+3=8

(i) $f(x) = |x|+|x-1|+|x-2|$ at the point x=0 and x=1

(ii) $f(x) = \begin{cases} \dfrac{x^4+4x^3+2x,}{\text{Sinx}}, & \text{if } x\neq 0 \\ 0, & \text{if } x=0 \end{cases}$

at the point x=0

4. (a) State and prove, Leibnitz's theorem for the n-th derivative of the product of two functions. 5

(b) If $y = \dfrac{\text{Sin}^{-1}x}{1-x^2}$, $|x|<1$, 5

show that

(i) $(1-x^2)y-3xy_1-y=0$

(c) Show that the series $\sum_{n=1}^{\alpha} \dfrac{1}{n^2+n}$, is a convergent series. 4

10. (a) Write lernma of rert test, ration test, limit comparison text-II. 6

(b) Examine whether the following series are convergent or divergent : 8

(i) $\dfrac{1}{2} + \dfrac{2}{2^2} + \dfrac{3}{2^3} + \ldots\ldots\ldots + \dfrac{n}{2^n} + \ldots\ldots\ldots\ldots$

(ii) $2 + \dfrac{3}{8} + \dfrac{4}{27} + \dfrac{5}{64} + \ldots\ldots\ldots\ldots$

(ii) $\dfrac{1}{3} + \left(\dfrac{2}{5}\right)^2 + \left(\dfrac{3}{7}\right)^3 + \left(\dfrac{4}{9}\right)^4 + \ldots..$

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(c) Define derivative of a function at any point c. 2

UNIT-IV

7. (a) Define convergent sequence. Show that the sequence $\left(\frac{3n+2}{n+1}\right)$, converges to 3. 5

(b) If (x_n) is a convergent sequence of real number's with xn>0, for all $n \in N$, and (x_n) converges to x, then show that x>0. 5

(c) Let (x_n) and (y_n) be sequence of real number's that converge to x and y respectively. Show that the sequence $(x_n y_n)$ converges to x.y. 4

8. (a) If $xn=\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+.....+\frac{1}{n(n+1)}$, then show that the sequence (x_n) is a convergent sequence. 5

(b) Define cauchy sequence. Show that the sequence $\left(\frac{1}{n}\right)$ is a cauchy sequence. 5

(c) Define monotone sequence with example. State monotone convergence theorem. 4

UNIT-V

9. (a) Define infinite series. Show that the series converges $\sum_{n=1}^{\alpha} \frac{1}{n(n+1)}$ to 1. 5

(b) Show that the series $\sum_{n=1}^{\alpha} \frac{1}{n}n$, diverges. 5

(ii) $(1-x^2)y_{n+2} - (2n+3)xy_{n+1} - (n+1)^2 y_n=0$

(c) Find the maximum and minimum value of the following expression : 4

(i) $5x^6-18x^5+15x^4-10$

(ii) $x^3-6x^2+24x+4$

UNIT-III

5. (a) State and prove Mean value theorem. 5

(b) Using $\in-\delta$ definition of derivative, prove the following identities : 3+3+3=9

(i) $\frac{d}{dx}(\mathrm{Sin}x)= \mathrm{Cos}x$

(ii) $\frac{d}{dx}(\phi(x) + \psi(x)) = \phi'(x) + \psi'(x)$

(iii) $\frac{d}{dx}(\tan x) = \mathrm{Sec}^2 x$

6. (a) Show that the function f(x), defined by- 6

$$f(x)=\begin{cases} x^2+x, & \text{if } 0<x<1, \\ 2, & \text{if } x=1 \\ 2x^3-x+1, & \text{if } 1<x<2 \end{cases}$$

show that f(x) is continuous at x=1, but it is not differentiable at x=1.

(b) A function f(x) is defined in 0<x<2 by

$$f(x) = \begin{cases} x^2+x+1, & \text{if } 0<x<1 \\ 2x+1, & \text{if } 1<x<2 \end{cases}$$

Examine the continuity and differentiability of f(x) at x=1. 6