PG Even Semester Examination, May 2025

MATHEMATICS

2nd Semester

Course No. : MAT - 554

Introduction to Graph Theory (IDC)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(All the symbols have their usual meanings)

(Answer one question from each unit)

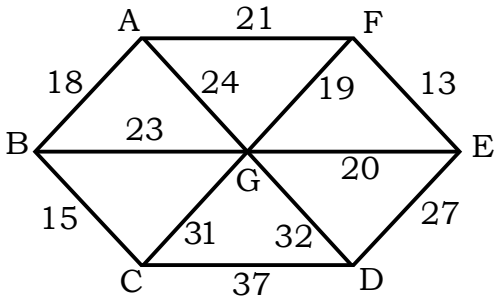
Unit - I

1. (a) Prove that for any simple graph G with six vertices, G or $\overline{G}$ contains a triangle. 3

   (b) Prove that in a simple graph, the number of vertices of odd degrees is even. 2

   (c) Prove that a simple graph is bipartite if and only if all its cycles are even. 3

   (d) Find the shortest spanning tree from the following connected graph using Prim's algorithm. 6



2. (a) If G is a tree with $\Delta \geq K$, then prove that G has atleast K vertices of degree one. 5

(b) Show that a graph G is a forest if and only if every edge of G is a cut edge. 2

(c) Prove that a vertex v of a tree G is a cut vertex at G if and only if $d(v)>1$ 3

(d) If a graph G contains a u-v walk, then prove that G-contains a u-v path. 4

Unit - II

3. (a) Prove that a connected graph G is an Eulergraph if and only if all vertices at G are of even degree. 4

(b) In a connected graph G with exactly 2k odd vertices, there exist k edge disjoint subgraphs, such that they together contains all edges of G. Prove that each subgraph is a unicursal graph. 5

Unit - V

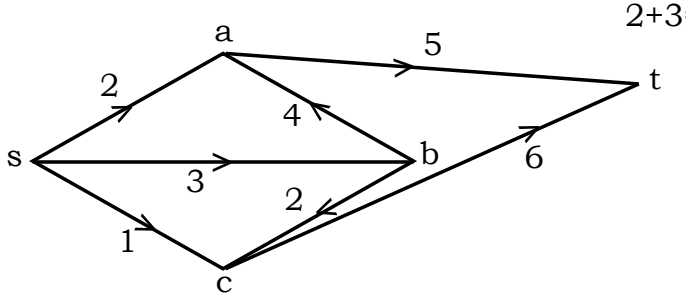
9. (a) Define :
Flow network, Edge-capacity, Source, Sink 2

(b) What is flow in a network? When a flow in a network is called a maximum flow? 1+1=2

(c) Let f be a flow and k be a cut such that $val\, f = cap\, k$. Show that f is a maximum flow and k is a minimum cut. 3

(d) Prove that for any flow and any cut (S, $\overline{S}$) of a network G, $val\, f = f^{+}(s) - f^{-}(s)$

10. (a) Find all the cuts of the following network and also find the capacity of the minimum cut 2+3=5



(b) State and prove Max-Flow-Min-Cut theorem. 1+6=7

(c) For any flow f and any cut k = (S, $\overline{S}$) in a network, prove that $val\, f \leq cap\, k$. 2

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(c) Prove that a graph is Eulerian if and only if it is connected and if the set of its edges can be partitioned into disjoint union of cycles. 5

4. (a) State and prove Ore's theorem. 1+6=7

(b) If G (V, E) be a simple graph for each $S \subseteq V(G)$, prove that $w(G-S) \leq |S|$ 3

(c) State Dirac's theorem. 1

(d) Show that a graph is Hamiltonian if and only if its closure is Hamiltonian. 3

Unit - III

5. (a) If G has atleast one edge, then prove that the sum of the co-efficients of its chromatic polynomial is zero. 3

(b) If the graph G is a tree with n vertices then find the chromatic polynomial of G. 4

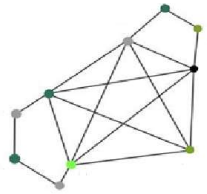
(c) Is $x^4 + 2x^3 - 4x^2$ a chromatic polynomial of a non-null graph. 3

(d) Define k-critical graph.
If G is k-critical, prove that $\delta \geq k-1$ 1+3=4

6. (a) If the maximum vertex degree of a graph is G, then prove that $\chi(G) \leq \Delta G+1$ 3

(b) Prove that a bipartite graph is 2-colourable. 2

(c) State Brook's theorem and hence find the chromatic number of the following graph - 1+3=4



(d) Show, by finding on appropriate edge colouring, that $\chi'(k_{m, n}) = \Delta k_{m, n}$

Unit - IV

7. (a) (i) Define planar graph. 1

(ii) Prove that for a connected plane graph with N vertices, E edges and F faces N - E + F = 2 4

(b) Prove that Petersen graph is non-planar. 5

(c) Prove that a planar graph is 4-colourable if its related triangular graph is 4-colourable. 4

8. (a) Show that there are atmost 5 platonic solids. 3

(b) State Jordan curve theorem and hence prove that k_5 is non-planar. 1+4=5

(c) Prove that the vertices of every planar graph can be properly coloured with five colours. 6