(b) Given IVP : $\frac{dy}{dx} = x + y$; $y(0) = 1$.

Find y (0.1), y (0.2) by using Runge-Kutta 4th order method. 4

Unit - V

9. (a) Classify the following PDES : 3

(i) $x^2 . f_{xx} + (1 - y^2) f_{yy} = 0$

(ii) $(1 + x^2) f_{xx} + (5 + 2x^2) f_{xy} + (4 + x^2) f_{yy} = 0$

(iii) $f_{xx} + 4 f_{xy} + (x^2 + 4y^2) f_{yy} = sin\,(xy)$

(b) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ such that

u (0, t) = 0, u (1, t) = 0, u (x, 0) = sin (πx) for $0<x<1$ and taking $h = \frac{1}{3}$, $k = \frac{1}{36}$ by

(i) Schmidth's method,

(ii) Crank-Nicolson method and compute only for 2 levels. 5

10. Solve $\frac{\partial^2 u}{\partial t^2} = t . \frac{\partial^2 u}{\partial x^2}$ subjec to the boundary contions

u (0, t) = 0 = u (4, t) and $\frac{\partial u}{\partial t}(x, 0) = 0$ and u (x, 0) = x (4 - x) by taking h=1 and obtain the solution upto 5 time steps. 8

★★★

PG Even Semester Examination, May 2025

MATHEMATICS

2nd Semester

Course No. : MAT - 555 (Theory)

(Numerical Analysis)

Full Marks : 40

Pass Marks : 16

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer five questions by taking one from each unit.

Unit - I

1. (a) Suppose n≥0 and f (x) is a real valued function and continuous on [a, b]. Show that there exists some point ξ∈ (a, b) such that 5

$$f(x) - P_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}\,\Pi_{n+1}(x), \quad \text{where}$$

$$\Pi_{n+1}(x) = (x - x_0)(x - x_1) \ldots . (x - x_n)$$

(b) Define absolute, relative and percentage errors in the numerical computation. 3

2. (a) State and prove existance and uniquness of a interpolating polynomial. 1+3=4

(b) Find the approximate value of y (2.5) for the function defined by the data :

x	0	1	2	3
$y = f(x)$	1	2	33	244

with $M_0 = M_3 = 0$, by using cubic-spine interpolation. 4

Unit - II

3. (a) Dervie 3-point derivative formula for aproximating the value of f′(x) at some nodal point, by the method of undetermined coefficients. Further, establish the error in this approximation. 5

(b) How large 'n' should be taken in order to ensure the error in the computation of $\int_0^1 e^x . dx$, by Composite Trapezoidal rule, is less than 10^{-8} ? 3

4. (a) Derive the Composite Simpson's Rule (CSR) for approximating $\int_a^b f(x) . dx$. Also, show that this rule is exact for polynomials of degree ≤ 3. 5

(b) Find 'n' such that the error in the approximation of $\int_0^1 \sin(\pi x) . dx$, by Composite Simpson's Rule; is less than 10^{-5}. 3

Unit - III

5. (a) Dervie the order of convergence for Newton-Raphson method. 4

(b) Given $f(x) = e^{-x} - x^2 = 0$, determine the smallest 'n' such that the root is approximated by Bisection method within an absolute error of 10^{-8}. 2

(c) Derive NRM formula to obtain the square root of the reciprocal of a positive integer N > 0. 2

6. (a) Show that the Secant method has a superlinear rate of convergence with an asymptotic error constant $\left| \frac{1}{2} \frac{f''(p)}{f'(p)} \right|$, where p is the exact root of $f(x) = 0$. 4

(b) Discuss the necessary conditions for convergence of the fixed point iteration method $x = g(x)$ where $g(x) = \frac{1}{2}\left(x + \frac{10}{x+4}\right)$ 2

(c) Find the approximate root of the equation $\cos x = 3x - 1$ correct to four decimal places, by using fixed-point iteration method. 2

Unit - IV

7. (a) Derive Modified Euler's method for solving the IVP : $\frac{dy}{dx} = f(x,y)\ ;\ y(x_0) = y_0$ 4

(b) Given IVP : $\frac{dy}{dx} = x - y^2\ ;\ y(0) = 1$.
Find y (0.2) correct to four decimal places by using Taylor's series method of order 4. 4

8. (a) Construct the finite-difference method, for approximating the solution of a BVP :
$\frac{d^2y}{dx^2} + f(x).\ \frac{dy}{dx} + g(x).y = r(x);$
$y(x_0) = a$, $y(x_n) = b$ where $f(x)$, $g(x)$, $r(x)$ are continuous functions on $[x_0 , x_n]$. Also, derive the error in this approximation. 4