8. (a) If there are sources at (±1, 0) and sinks at (0, ±1) all of equal strength. Find complex potential function for the flow. Also, obtain the speed of fluid particles at any point. 4

(b) Determine the average velocity distribution, shear stress & coefficient of shear stress on the wall following pressure driven steady flow through a long cylindrcial pipe. 8

(c) Model a fluid dynamics equation with conditions in case of a flow through a straight channel, whose lower plate is in uniform motion U_0 (say). 2

Unit - V

9. (a) What is Prandtl's boundary layer theory? Derive two dimensional boundary layer equations for flow over a plane wall, using order of magnitude approach. Also, reduce them for steady case. 2+6+1=9

(b) A polynomial of 2^{nd} degree is assumed for a flow field in the boundary layer. Obtain the velocity for the flow. 5

10. (a) For boundary layer flow over a flat plate, obtain Blasius equation. 8

(b) A velocity distribution in the boundary layer over a flat plate is approximated by $\mu = U_{\infty} (1 - e^{-\eta})$, where, $\eta = y/\delta$ (say). Obtain the skin-friction at the plate. 6

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PG Even Semester Examination, May 2025

MATHEMATICS

4th Semester

Course No. : MAT - 652

(Mechanics & Applied Fluid Dynamics)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer one question from each unit)

Unit - I

1. (a) What are constraints? What are various classifications of constraints? Cite an example in each case with brief introduction. 8

(b) Define a Lagrangian of a system. For the Lagrangian:
$L(q_1, q_2, \dot{q}_1, \dot{q}_2) = \dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_1\dot{q}_2 - q_1^2 - q_2^2$, find the conjugate momenta and hence construct the corresponding Hamiltonian. 1+5=6

2. (a) What is Hamilton's Action Principle (HAP)? Establish its invariance under generalized co-ordinate transformation. 2+4=6

(b) The Hamiltonian of a dynamical system is given by,
$H(q_1, q_2, p_1, p_2) = q_1 p_1 - q_2 p_2 - aq_1^2 + bq_2^2$;

where, a, b are any constants. Solve the problem. 8

Unit - II

3. (a) What are canonical co-ordinates & canonical transformations? Is the transformation defined by:

$$q\,(Q,P) = \sqrt{\frac{P}{\sqrt{K}}}\;Sin\,\theta \;\;\&$$

$$p\,(Q,P) = \sqrt{m\,P\,\sqrt{K}}\;Cos\,\theta.$$

canonical? Justify. 2+5=7

(b) Define Poission's bracket of two dynamical variables. Establish that Poission's bracket is invariant under a set of canonical transformations. 1+6=7

4. (a) What is a generating function? A transformation $Q\,(q,p) = \sqrt{q}\,\cos 2p$ & $P\,(q,p) = \sqrt{q}\,\sin 2p$, which is canonical; show that there exists a generating function of the form : $F_1\,(q,Q) = \frac{q}{2}\,cos^{-1}\left(\frac{Q}{\sqrt{q}}\right)$ 1+5=6

(b) Find the position x(t) & momentum p(t) at any time for the motion of a particle given by Hamiltonian :

$$H\,(q,p) = \frac{p^2}{2m} + \frac{1}{2}\,mw^2n^2$$ 8

Unit - III

5. (a) Differentiate between Lagrangian and Eulerian description for fluid motion. Can it be possible to change one form to another? Justify. 2+4=6

(b) What are stream lines and path lines? Write the condition for which they represent the same line. 2+1=3

(c) A 3-D flow field is given by $\left(\frac{3xz}{r^5}\;,\frac{3yz}{r^5}\;,\frac{3z^2-r^2}{r^5}\right)$

Is the liquid motion possible? If so, find the velocity potential function of the flow. 2.5+2.5=5

6. (a) A 2-D flow field is specified by $\vec{q} = \frac{\lambda\,(x\hat{j} - r\hat{\imath})}{x^2+y^2}$, λ = constant. Is it represent a possible motion of an incompressible fluid? If so, determine the equation of the stream lines. Is the motion a potential kind? If so, determine the velocity potential. 2+2+1+2=7

(b) Show that the velocity field

$$\vec{q} = \left(\frac{A\,(x^2-y^2)}{(x^2+y^2)^2}\;,\quad \frac{2A\,xy}{(x^2+y^2)^2}\;,0\right)$$

satisfied the equation of motion for inviscid, incompressible flow. 7

Unit - IV

7. (a) Establish that the current function always exists in all type of two-dimensional motion whether rotational or irrotational. Also, show that for a 2-D, potential type of flow the current function satisfies Laplace equation. 2+2=4

(b) What are sources, sinks and doublets? In liquid bounded by the axes of x and y in the first quadrant, there is a source of strength 4+m' at a distance 'r' from the origin on the bisector of the ∠XOY. Prove that, the complex potential is - m log (z^4 + a^4). 3+5=8

(c) What are Newtonian and non-Newtonian fluids? Cite examples in each case. 2