(b) Show that for every prime p and every positive integer n, there exists a finite field with p^n elements. 7

(c) Find all subfields of a field with 125 elements. 3

10. (a) Show that any finite extension of a finite field is a Galois extension. 4

(b) Let $\mathbb{F}_{p^n}$ denote the finite field with p^n elements. Show that $\mathbb{F}_{p^m}$ is a subfield of $\mathbb{F}_{p^n}$ iff m/n. 6

(c) Find the Galois group of $x^3 - 2$ over Q. 4

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PG (CBCS) Even Semester Examination, May 2025

MATHEMATICS

2nd Semester

Course No. : MTMCC - 202 (O)

(Algebra - II)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer one question from each unit)

Unit - I

1. (a) Show that any finite integral domain in a field. 5

(b) Show that characteristic of a field is either 0 or a prime number. 5

(c) Show that every maximal ideal in a commutative ring with unity is a prime ideal. 4

2. (a) Show that order of any finite integral domain in power of some prime. 4

(b) Find all prime ideals of $\mathbb{Z}$, the ring of integers. 4

(Turn Over)

(c) Let R be a commutative ring with unity and M be any proper ideal of R. Show that M is a maximal ideal of R iff R/M is a field. 6

Unit - II

3. (a) Let R be a field. Show that R[x] is a principal ideal domain. 5

(b) Show that every prime element in an integral domain in irreducible. Give an example to show that the converse is not true. 2+3=5

(c) Is $\mathbb{R}$ [x, y] a P.I.D? Justify. 4

4. (a) Let R be a factorisation domain. Show that R is a UFD iff every irreducible element is prime. 6

(b) Show that ascending chain condition holds for ideals in a P.I.D? Does descending chain condition holds for ideals? Justify. 5+3=8

Unit - III

5. (a) State and prove Eisensteins irreducibility criterion. 2+6=8

(b) Show that every finite field extensioin is algebraic. Is the converse true. Justify your answer. 3+3=6

6. (a) Let K|F be a field extension. Show that a∈k is algebraic over F iff F(a)|F is finite. 3+3=6

(b) Show that doubling a cube is not possible using ruler and compass only. 4

(c) Show that x^4+1 is irreducible over Q. 4

Unit - IV

7. (a) Let F be a field and p(x) be any non constant polynomial over F. Show that there exists an extension E of F with [E :F] ≤ n ! in which p(x) has a root. 8

(b) Define normal field extension. Construct a normal field extension of Q and another extension of Q which is not normal. 2+2+2=6

8. (a) Find the splitting field of $1+x+x^2+x^3+x^4$ over Q. 5

(b) Show that any field extension of a field of characteristic 0 is separable. 5

(c) Give example of a inseparable field extension. 4

Unit - V

9. (a) Show that a finite field can't have characteristic 0. 4