PG (CBCS) Even Semester Examination, May 2025

MATHEMATICS

2nd Semester

Course No. : MTMCC - 204 (O)

Discrete Mathematics (Open-II)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(All the symbols have their usual meanings)

(Answer one question from each unit)

Unit - I

1. (a) Show that every square integer is of the form 5q, (5q+1), (5q-1) 4

   (b) Show that (a + b, a - b) = 1 if (a, b) = 1 and one of a and b is even and the other is odd. 4

   (c) Find the general solution of $8x+7y = 89$ 6

2. (a) The product of any three consecutive integers is divisible by 3! 4

   (b) Explain the Pigeon hole principle. 3

   (c) Establish that the difference of two consecutive cubes is never divisible by 2. 3

   (d) If $k>0$, then prove that $(ka, kb) = k\,(a, b)$ 4

(Turn Over)

Unit - II

3. (a) Show that there exists infinitely many primes. 4

(b) State and prove Wilson's theorem. 1+4=5

(c) Define reduced system of residues and show that { 22, - 1, 43, 46, - 19, 79, 113, 452 } is a RRS (mod 15) 5

4. (a) If p is a prime then show that 3
$(a + b)^p \equiv (a^p + b^p) \pmod p$

(b) Find the least residue of $7^{973} \pmod{72}$ 3

(c) Solve, $59x \equiv 31 \pmod{67}$ 4

(d) State and prove Fermat's little theorem. 1+3=4

Unit - III

5. (a) Find all the primitive roots of 17. 5

(b) Find the solutions of $x^{15} \equiv 7 \pmod{19}$ 4

(c) Solve $3y^2 + 2y + 3 \equiv 0 \pmod{11}$ by using quadratic reciprocity law. 5

6. (a) Find the quadratic residues of 29. 4

(b) State and prove Möbious inversion formula. 6

(c) Find $\tau(n)$ and $\sigma(n)$ for $n = 28$ 4

Unit - IV

7. (a) Define isomorphism in a graph with example. 3

(b) Define spanning tree. Show that every connected graph has a spanning tree. 1+5=6

(c) Show that a graph is a tree if and only if it is minimally connected. 5

8. (a) Show that the maximum number of edges in a complete bipartite graph of n vertices is $n^2/4$. 5

(b) Prove that in a non-directed graph, the number of vertices of odd degree is even. 5

(c) Prove that a graph is bipartite if and only if all its cycles are even. 4

Unit - V

9. (a) State and provel Euler's formula. 1+5=6

(b) For what value of n is the graph k_n Eulerian? Find the value of m, n for which km, n are Euler graph? 2+2=4

(c) (i) Define Hamiltonian graph? Give example. 1+1=2

(ii) State Kuratowski's theorem. 2

10. (a) Define planar graph. Prove that k_5 and $k_{3,3}$ are non-planar. 1+3+3=7

(b) Explain, why the Konigsberg bridge problem has no solution. 2

(c) Prove that a simple graph with n vertices (n>2) is Hamiltonian if the sum of the degrees of every pair of non-adjacent vertices is atleast n. 5

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