(b) Solve $\nabla^2 u = 0$, $0 \le x \le a$, $0 \le y \le b$ satisfying $u(0, y) = 0$, $u(x, 0) = 0$, $u(x, b) = 0$ and

$$\frac{\partial u}{\partial x}(a, y) = T.\ \mathrm{Sin}^3\left(\frac{\pi y}{a}\right)$$ 7

Unit - V

9. (a) Prove that the general solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 . \frac{\partial^2 u}{\partial x^2}$ is

$u = f(x - ct) + g(x + ct)$ 7

(b) Obtain the periodic solution of wave equation in the form. $u(x, t) = A.\ e^{i\,(kx = wt)}$ where $i = \sqrt{-1}$, $k = \pm w/c$, A is constant and hence define various terms involved in wave propogation. 7

10. (a) Obtain the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 . \frac{\partial^2 u}{\partial x^2}$ under the following conditions : 1+3+3=7

(i) $u(0, t) = u(2, t) = 0$

(ii) $u(x, 0) = \mathrm{Sin}^3\left(\frac{\pi x}{2}\right)$

(iii) $\frac{\partial u}{\partial t}(x, 0) = 0$

(b) Solve the initial value problem. 7

$$\frac{\partial^2 u}{\partial t^2} - c^2 . \frac{\partial^2 u}{\partial x^2} = e^x$$

with given data $u(x, 0) = 5, \frac{\partial u}{\partial t}(x, 0) = x^2$

★★★

PG (CBCS) Even Semester Examination, May 2025

MATHEMATICS

2nd Semester

Course No. : MTMCC - 205 (O)

(Partial Differential Equations)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer one question from each unit)

Unit - I

1. (a) Show that $z = ax + (y) + b$ is complete integral of pq=1 and show that it has no singular solution. 3+2=5

(b) Find the integral of

$$yz.dx + 2xz.\ dy - 3xy.dz = 0$$ 5

(c) Show that the euqations $f = xp - yq - x = 0$ and $g = x^2p + q - xz = 0$ are compatible and find a one parameter family of common solution. 4

2. (a) Find the integral surface of $xp + yq = z$ containing the circle defined by $x^2 + y^2 + z^2 = 4$, $x+y+z = 2$ 5

(b) Find the complete integral of $p^2x + q^2y = z$ by Jacobi's method. 5

(c) Find the complete integral of $f = z^2 - pq\,xy = 0$ by Charpit's method. 4

Unit - II

3. (a) Find the surface passing through $x^2 + y^2 = a^2$, $z=0$ and satisfying $S = 8xy$; where $S = \frac{\partial^2 z}{\partial x^2 y}$ 4

(b) Reduce

$$\frac{\partial^2 z}{\partial x^2} + 2.\frac{\partial^2 z}{\partial x^2 y} + \frac{\partial^2 z}{\partial y^2} = 0$$

to its canonical form and hence solve it. 6

(c) Find the characteristics of 4

$$(\mathrm{Sin}^2 x)\frac{\partial^2 u}{\partial x^2} + (2\,\mathrm{Cos}\,x)\frac{\partial^2 u}{\partial x^2 y} - \frac{\partial^2 u}{\partial y^2} = 0$$

4. (a) Find the families of characteristics of $(1-x^2)\,u_{xx} - u_{yy} = 0$ in elliptic and hypterbolic cases. 7

(b) Solve

$$x^2.\frac{\partial^2 u}{\partial x^2} + 2xy.\frac{\partial^2 u}{\partial x^2 y} + y^2.\frac{\partial^2 u}{\partial y^2} = 0$$

by Monge's method. 7

Unit - III

5. (a) Using Dubamel's principle to solve the IBVP:

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = f(x,t);\ 0 < x < L, \quad 0 < t < \infty$$ with boundary conditions u (0, t) = u (L, t) = 0 and initial conditions u $(x, 0) = 0$, $0 \le x \le L$ 7

(b) Solve the one dimensional heat equation in the region $0 < x \le \pi$, $t > 0$ subject to the conditions:

(i) T remains finite as $t \to \infty$

(ii) T = 0 if $x = 0, \pi\ \forall\ t$

(iii) At t = 0 ; 7

$$T = \begin{cases} x & ;\ 0 < x < \frac{\pi}{2} \\ \pi - x & ;\ \frac{\pi}{2} < x < \pi \end{cases}$$

6. (a) Solve the heat equation described by

$$\frac{\partial u}{\partial t} = k.\frac{\partial^2 u}{\partial x^2};\ -\infty < x < \infty,\ t > 0$$

subject to $u(x, 0) = f(x)$; $-\infty < x < \infty$ 7

(b) At infinite one-dimensional solid defined by $-\infty < x < \infty$ is mentioned at zero temperature initially. There is a heat source of strength $g_s(t)$ units situated at $x = \xi$ which releases constant heat continuously for t>0. Find an expression for temperature distribution in the solid for t>0. 7

Unit - IV

7. (a) Show that the solution of a Dirichlet problem, if exists, for bounded region is unique. 6

(b) Find the solution of the following Helmholtz equation 8

$$\nabla^2 u + k^2 u = 0$$

8. (a) Show that the Laplace's equation $\nabla^2 u$ takes the form $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$. Also, obtain the most general solution. 7

(Turn Over)