PG (CBCS) Even Semester Examination, May 2025

MATHEMATICS

4th Semester

Course No. : MTMCC - 403 (O)

(Fluid Dynamics)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer one question from each unit)

Unit - I

1. (a) What are different methods of description of fluid motion? Also discuss the methods. 4

(b) What are material, local and convective derivatives? Find an operator form of relation relating them. 5

(c) Given a field of the form :
$\vec{q} = (10\,x^2 y\,, 15xy\,, 25t - 3xy)$
Find the acceleration of a fluid particle at a point (1, 2, -1) at time t = 0.5s 5

2. (a) What is equation of continuity in fluid system? Obtain the vector form of equation of continuity. 6

(b) Determine the constants *l, m, n* in order that the velocity, $\vec{q} = \frac{(x+lr)\,\hat{\imath} + (y+mr)\,\hat{s} + (z+nr)\,\hat{R}}{r(x+r)}$ where, $r = \sqrt{x^2+y^2+z^2}$ many satisfy the equation of continuity for a liquid motion. 8

(Turn Over)

Unit - II

3. (a) What is Lagrange's stream function? Establish that the difference of the values of stream function at two points represents the flux of a fluid across any curve joining the two points. 2+4=6

(b) State & prove the theorem of Blasius. 8

4. (a) What are 2-D sources, sinks and doublets?A complex potential for a 2-D flow is given by

$$w(z) = log\left(z - \frac{a^2}{z}\right), z \in \mathbb{C}$$

obtain the velocity potential function, stream function & streamlines of flow. 6

(b) What is circle theorem? Use this theorem to find the image of a doublet with respect to a circle. 8

Unit - III

5. (a) What do you mean by three dimensional motion? Use analytical method to find the general solution for continuity in case of irrotational motion near a spherical object. 6

(b) For a liquid streaming past a fixed sphere, obtain the lines of flow relative to the sphere. 8

6. (a) What is vorticity? Show that the product of the cross sectin & the vorticity at any point on a vortex filament is constant along the filament. 6

(b) What is Karman's vortex street? Obtain velocity potential function & stream function at any point of the system. Also show that under this set up, the vortex at (0, 0) moves with the velocity $\frac{k}{2a} \tan h\left(\frac{\pi b}{a}\right)$ where a & b are the spacing of vortices & vortex rows respectively. 8

Unit - IV

7. (a) What is viscosity? State & establish Newton's law of viscosity. What do you mean by Newtonian & non-Newtonian fluids? Give example in each case. 6

(b) Derive Navier-Stokes equations for the flow of Newtonian, incompressible, constant viscous fluid. 8

8. (a) Find the velocity profile of a Newtonian, viscous, incompressible fluid through a straight channel under a resonable pressure gradient effect. 6

(b) Discuss Hagen-Poiscuille flow in regard to volumetric flow rate, average velocity & skin-friction on the wall. 8

Unit - V

9. (a) What is Prandtl's boundary layer theory? Obtain the boundary layer equations by any suitable approach. 2+7=9

(b) Consider a laminar boundary layer on a flat plate with a velocity profile given by $u = U_\infty\left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right)$, *where* $\eta = \frac{y}{8}$

If $\delta = \frac{4.64\,x}{Rex}$, determine the skin-friction on the plate. 5

10. (a) Obtain Blasius equation for flow over a flat plate. 8

(b) Find momentum integral equation for two dimensional incompressible laminar boundary layer. 6

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