PG (CBCS) Even Semester Examination, May 2025

MATHEMATICS

4th Semester

Course No. : MTMCC - 404(B)

(Differential Geometry)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(All the symbols have their usual meanings)

(Answer one question from each unit)

Unit - I

1. (a) Find the curvature and torsion of the curve r = (cos 2u, sin 2u, 2 sin u) 5

   (b) Prove that the length of the curve is invariant under parametric transformations. 4

   (c) State and prove Serret-Frenet formulae. 5

2. (a) Find the equation of the osculating plane at a general point on the cubic curve given by r = (u, u^2, u^3) and show that the osculating planes at any three points of the curve meet at a point. 6

   (b) Derive the curvature and torsion of the spherical indicatrix of the tangent to a space curve γ. 5

   (c) Prove that the binormals at the two corresponding

(Turn Over)

points of a pair of Bertrand curves will include the same angle as that between tangents. 3

Unit - II

3. (a) Prove that the osculating plane at any point on the edge of regression is the tangent plane to the developable at that point. 6

(b) Obtain the surface equation of a cone with semi-vertical angle α and find the singularities, parametric curves, tangent plane at a point and surface normal. 8

4. (a) Find the envelop of the plane

$$\frac{x}{a}\cos\theta\sin\phi+\frac{y}{b}\sin\theta\sin\phi+\frac{z}{c}\cos\phi=0$$

where θ and ϕ are parameters. 6

(b) Prove that the tangents to the edge of regression are the characteristic lines of the developable. 7

(c) State the fundamental theorem for space curve. 1

Unit - III

5. (a) Prove that the first fundamental form of a surface is a positive definite quadratic form in du and dv. 4

(b) If ω is the angle between the parametric curves at the point of intersection, then prove that $\tan\omega=\frac{H}{F}$ 4

(c) Find the orthogonal trajectories of the parametric curve u = constant on the surface $r = (u+v,\ 1-uv,\ u-v)$ 6

6. (a) State and prove Meusiner's theorem. 1+5=6

(b) Define normal curvature and hence prove that if k_n be the normal curvature at a point on the surface then $k_n=\frac{I}{II}$ 1+7=8

Unit - IV

7. (a) Find the Gaussian curvature of $r(u, v) = (u\cos v,\ u\sin v,\ \cos 2v)$ 4

(b) Show that all points on a sphere are umbilics. 6

(c) Show that $e^z\cos x = \cos y$ is minimal surface. 4

8. (a) Prove that the Dupin indicatrix at a point on a sphere is a circle. 6

(b) Find the nature of the points on the surface $r(u, v) = (u,\ v,\ u^2 - v^2)$ 2

(c) For the surface $r = (u\cos v,\ u\sin v,\ f(u))$, find the principal directions and principal curvatures at any point on the surface. 6

Unit - V

9. (a) Prove that the helix on a cylinder is a geodesic and conversely. 7

(b) Show that a curve on a sphere is a geodesic if and only if it is a great circle. 7

10. (a) State and prove the necessary and sufficient condition for a curve $u = u(t)$ and $v = v(t)$ on a surface $r = r(u, v)$ to be a geodesic. 1+5=6

(b) Show that the curves $u+v$ = constant are geodesic on a surface with the metric $(1+u^2)\,du^2 - 2uv\,du\,dv + (1+v^2)\,dv^2$ 8

★★★