characteristic roots of the variance-covariance matrix Σ. 7

8. (a) Write the assumptions of the multivariate analysis of variance. Explain the one-way MANOVA with necessary deduction. 2+5=7

(b) What is Factor Analysis? Explain the process of estimating of the factor scores. 2+5=7

Unit - V

9. (a) What is Cluster Analysis? Explain some applications of cluster analysis in different fields of knowledge. 2+5=7

(b) Give a brief note on how cluster are formed using hierarchical clustering. 7

10. (a) What is lazy learning? What is a classification model? Discuss the advantages and limitations of k-nearest neighbour and algorithm for classification. 1.5+1.5+4=7

(b) Describe the working principle of the Naive Bayesian classifier-explaining clearly the assumptions and suitability of the model. 3+4=7

★★★

PG Even Semester Examination, May 2025

STATISTICS

4th Semester

Course No. : STS - 652

Multivariate Analysis (Core)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer five questions taking one from each unit)

Unit - I

1. (a) With the help of examples explain how multivariate data analysis can be used in the study of (i) Medicine and (ii) Sports. 6

(b) Explain the quadratic form in the multivariate normal density function.
Suppose u_1 and u_2 are two independent p-dimentional variables such that $U = U_1 + U_2$ and $U \sim N_p(\mu, \Sigma)$. Then show that U_1 and U_2 are also p-dimensional normal variates. 2+6=8

2. (a) Let $X_1, X_2, \ldots X_n$ be independently distributed random variables X_i, having distribution $N(\beta+\nu z_i, \sigma^2)$ where z_i is a given number $i = 1, 2, \ldots, n$ and

$\Sigma z_i = 0$, then

(i) Find the distribution of $(X_1, X_2, \ldots, X_n)$

(ii) Find the distribution of $\overline{X}$ and $g = \frac{\Sigma Z_i X_i}{\Sigma Z_i^2}$ for $z_i^2 > 0$.

4+4=8

(b) If $(X_1, X_2, \ldots, X_p)$ follows multinomial distribution then deduce the regression of X_i on X_j and hence interpret the result. 6

Unit - II

3. (a) Define the Wishart distribution. Deduce the characteristic function of Wishart distribution.

2+5=7

(b) Deduce an expression for the multiple correlation coefficient between X_i and $\beta \underset{\frown}{X}^{(2)}$, where X_i and $\beta \underset{\frown}{X}^{(2)}$ have usual meanings. 7

4. (a) Explain the multivariate general linear model and hence deduce that $\hat{U}$ and $\hat{B}$ are independent of each other - where $\hat{U}$ and $\hat{B}$ have usual meanings.

7

(b) Explain the concept of Canonical Correlation. Show that for two partitioned sub-vector $\underset{\frown}{X}^{(1)}$ and $\underset{\frown}{X}^{(2)}$ of vector $\underset{\frown}{X}$ having variance-covariance matrix Σ i.e. partitioned as

$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$ the characteristic equation is given by -

$|\lambda^2 \, T \, \Sigma_{11}^{-1} \, \Sigma_{12} \, \Sigma_{22}^{-1} \, \Sigma_{21}| = 0_n$ 7

Unit - III

5. Deduce the likelihood ratio criterion for testing the equality of the variance-covariance matrices and for testing the equality of mean vectors and variance-covariance matrices. 14

6. (a) Deduce the test statistic for two sample problem of Hotelling's T^2 for testing the null hypothesis $H_0 : \underset{\frown}{\mu_1} = \underset{\frown}{\mu_2}$ when the variance-covariance matrix Σ is common but unknown. 8

(b) Define the Mahalonobis D^2 statistic. Show that the statistic is invariant under non-singular linear transformation. 2+4=6

Unit - IV

7. (a) Deduce Fisher's discriminating procedure for performing discriminant analysis. 7

(b) What is the role of principle components analysis in multivariate analysis? Deduce an expression for the first principal component based on the