Unit - V

9. Develop a general method for obtaining confidence intervals. Obtain a 100 (1-α)% confidence interval for large sample size for the parameter θ of the Poisson distribution. 8+7=14

10. Discuss the problem of interval estimation. Obtain the shortest confidence interval for the variance of a normal population with unknown mean based on a random samle of size n. 4+10=14

★★★

PG (CBCS) Even Semester Examination, May 2025

STATISTICS

2nd Semester

Course No. : STSCC - 204

(Statistical Inference-I)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer five questions taking one from each unit)

Unit - I

1. (a) Define unbiased estimator with a simple example. If $(X_1, X_2, ..., X_n)$ is a random sample from a normal population N (μ, 1). Show that $T = \frac{1}{n}\sum_{i=1}^{n} x_i$ is an unbiased estimator of μ^2+1 2+5=7

   (b) State and prove the invariance property of consistent estimator. 7

2. (a) What do you mean by sufficiency? Let $(x_1, x_2, ..., x_n)$ be a random sample from N (μ, σ^2) population. Using Fisher-Neyman factorization criteria find sufficient estimators for μ and σ^2. 7

(b) Explain the role of order statisfies in statistical inference. If $(X_1, X_2, ..., X_n)$ is jid U(a, b) variates, then find the sufficient statistics for estimating a and b. 2+5=7

Unit - II

3. (a) Define minimum variance unbiased estimator. Show that minimum variance unbiased estimator are unique. 2+5=7

(b) A random sample $(x_1, x_2, ..., x_n)$ is taken from a normal population with mean zero and variance σ^2. Examine if $\sum_{i=1}^{n} x_i^2/n$ is a minimum variance bound estimator for σ^2. 7

4. (a) State and prove Rao-Cramer's inequality. 10

(b) Let $(x_1, x_2, ... , x_n)$ be a random sample from $\cup(0, \theta)$ population. Obtain minimum variance unbiased estimator for θ. 4

Unit - III

5. (a) If a sufficient estimator exists, prove that it is a function of the maximum likelihood estimator. 7

(b) Explain maximum likelihood estimator for estimating parameters of a distribution. What are

the important properties of maximum likelihood estimator? 5+2=7

6. (a) Describe method of moments for estimating parameters of a distribution. Under what conditions, the maximum likelihood method and method of moments will provide the same estimator? 5+2=7

(b) Find the maximum likelihood estimator and the method of moment estimator of θ based on random sample $(X_1, X_2, ..., X_n)$ from the population having density function. 4+3=7

$$f(x : \theta) = (1+\theta)\, x^{\theta} ; \quad 0<x<1 ; \quad \theta>0$$

Unit - IV

7. State and prove Neyman pearson lemma for testing simple null hypothesis against simple alternative hypothesis. 14

8. (a) Define null hypothesis, best critical region and level of significance. 2+2+2=6

(b) A random sample of size n is taken from $N(\theta, 100)$ distribution. Find the best critical region for testing $H_0: \theta = 70$ aginst $H_1: \theta = 73$. 8