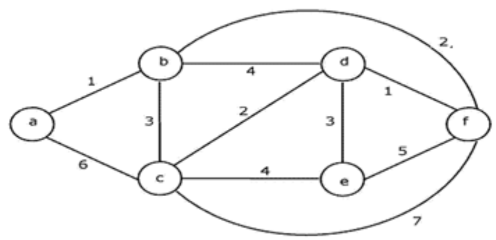
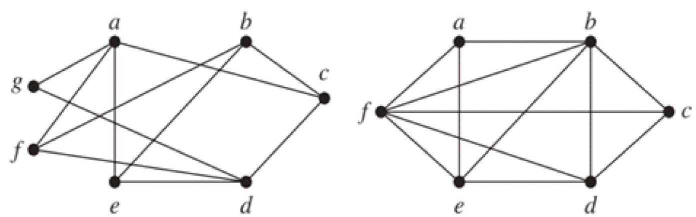
(c) Explain Breadth First Search and Depth First Search with an example (4)

10. (a) Define Eulerian and Hamiltonian graphs with an example. (5)

(b) Find the mininum spanning tree using Krushal'sAlgorithm and also find the minimum cost. (5)



(c) Determine whether C_6 and K_3 graph are bipartite graph?Are the graphs G and H displayed in figure below bipartite? (4)



G H

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B.Tech Even Semester Examination, May, 2025

Computer Science & Engineering

(4th Semester)

Course No: CSE-401

(Discrete Mathematics and Graph Theory)

Full Marks: 70

Pass Marks: 28

Time: 3 hours

Note:

1. Attempt any one from each unit.
2. Begin each answer in a new page
3. Answer parts of a question at a place
4. Assume reasonable data wherever required
5. The figure in the right margin indicates full marks for the question
6. All the mathematical symbols and abbreviations have their usual meanings.

UNIT-I

1. (a) Let Z be the set of integers and R be the relation on Z defined by R= {(x, y): x, y $\in$ Z and x-y is divisible by 3}. Determine the equivalence classes generated by the elements of Z. (5)

(b) Prove using laws of logic: A- (B∪C) =(A-B) $\cap$ (A-C) and verify the result with a Venn diagram. (5)

(c) Let N be the set of all natural numbers. Prove that relation on N defined by (a, b) R (c, d) iff a + d= b + c is an equivalence relation. (4)

2. (a) Prove that: $B\cup\left(\bigcap_{i=1}^{m} A_i\right)=\bigcap_{i=1}^{m}\left(B\cup A_i\right)$ (5)

(b) Let S be the set of rational numbers under the operation multiplication and the relation R= (a/b) R (c/d) iffa.d =b.c. Show that R is equivalence relation on S. (4)

(c) Let X be the set of factors of a particular positive integer m and let the relation R be divides relation. Draw the hasse diagram for the above poset for m=12. From the Hasse diagram, find out the maximal and minimal element. (5)

UNIT-II

3. (a) What do you mean by Semigroup and Monoid. Explain with an example? (5)

(b) Let f, g and h be mapping from N to N when N is the set of naturals such that

f(n)=n+1,g(n)= 2n, h(n)= $\begin{Bmatrix} 0\text{,n is even} \\ 1\text{,n is odd} \end{Bmatrix}$

(i) Show that f, g, and h are functions

(ii) Determine fog, hog, and (fog) oh, where ‘o’ stands for the composition of functions. (3+3)

(c) For the function

f (x) = y = 2x + 1, find the range when domain = {-3, -2, -1,0,1,2,3} Let A = {1, 2} and B = {3, 6} and f and g be functions from A to B, defined by f(x) = 3x and g(x) = x^2 + 2. Show that f = g. (3)

4. (a) Prove that F: R → R defined by F (x) = x2 +3 is neither **one-one** nor **onto function** (4)

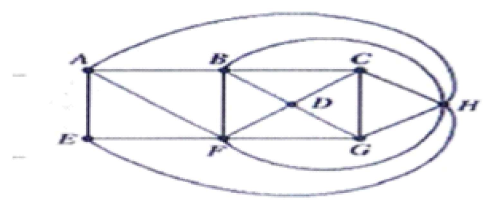
(b) Explain with an example of the following terms:(a)Wheel Graph (b) Fundamental Cut Sets and Cut Vertices (2*3)

(c) Let δ and Δ denotes the minimum and maximum degree of a simple graph. And also, e and V represent the number of edges and vertices of a graph. Prove that $\delta \leq \frac{2e}{V} \leq \Delta$. (2)

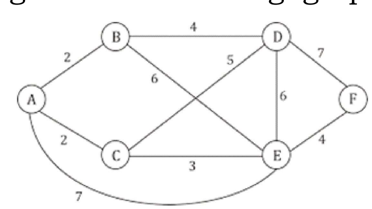
(d) Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph, then the number of bounded faces in any embedding of G on the plane is equal to? (2)

UNIT-V

9. (a) Use the Welsh- Power algorithm to determine a vertex coloring for the graph. Also, show the trace of the vertex colouring. Whether Welsh Power algorithm always give anexact solutions. If not, give a reason for your answer. (5+1)



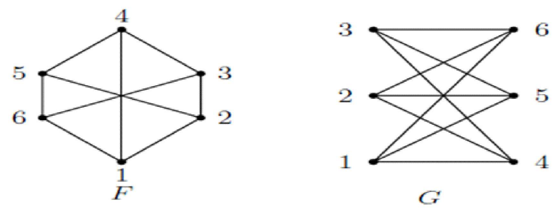
(c) What do you mean by Matching in a graph? Find any two edge covering and vertex covering of the following graph? (5)



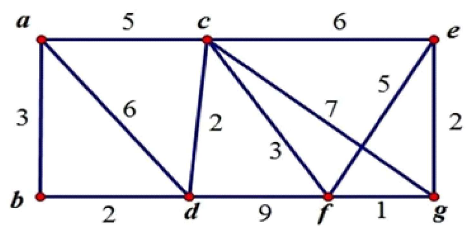
(c) Use mathematical induction to show that "5^n -1 is divisible by 4" for all $n \geq 1$ (5)

UNIT-IV

7. (a) Define isomorphism between two graphs. And also determine whether the following graphs are isomorphic. If yes, justify your answer. (2+2)



(b) Find the shortest path from a to g by Dijkstra's Shortest Path Algorithm. (5)



(c) Define vertex connectivity and edge connectivity. Give the relationship between them. G is a graph with 11 edges and 7 vertices. What is the maximum value of vertex connectivity? (5)

8. (a) The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences cannot be the degree sequence of any graph? Use Havel Hakemi Procedure.

(I) 7, 6, 6, 4, 4, 3, 2, 2
(II) 8, 7, 7, 6, 4, 2, 1, 1 (2+2)

(b) Find the domain of each of the following functions:

(i) y= + x - 2 (ii) y= + (2 - x) (4 + x) (2.5+2.5)

(c) Prove thata set of positive integers (excluding zero) with multiplication operation is a monoid. (5)

UNIT-III

5. (a) Translate the following statement into symbolic form using quantifiers and write the symbolic form, and negate the following statements:

(i) Some people are not admired by everyone.

(ii) Everyone should help his neighbours, or his neighbours will nothelp him. (2+2)

(b) Verify the validity of the following arguments: "If I study, then I will not fail in mathematics. If I do not play hockey, then I will study. But I failed in mathematics. Therefore, I must have played hockey". (6)

(c) Suppose the average of 4 different integers is 20. Prove by the method of contrapositive that one of the integers is greater than 21. (4)

6. (a) Translate the following statement into symbolic form using quantifiers and also negate the quantified statement.

"All boys in the class are taller than all the girls" (5)

(b) Show that $[(p \to q) \wedge (q \to r)] \to (q \to r)$ is a tautology. (4)