2017/ODD/18/24/MCS–302/356

UG Odd Semester (CBCS) Exam., December—2017

COMPUTER SCIENCE

(3rd Semester)

Course No. : MCSCC–302

(Mathematics—II)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* By using definition of limit, show that

$$\operatorname*{Lt}_{x \to 2} \frac{x^3 - 4}{x^2 + 1} = 4/5$$ 5

(b) If $f : A \to R$ is a function, where $A \subseteq R$, and if c is a cluster point of A, then show that $\operatorname*{Lt}_{x \to c} f(x)$ is unique. 5

(c) Define bounded function and monotonic function with an example each. 4

2. *(a)* Evaluate the following limits : 2+2+2+3=9

(i) $\operatorname*{Lt}_{x \to 0} \dfrac{\sin x - \tan x}{x^3}$

(ii) $\operatorname*{Lt}_{x \to 0} \dfrac{1 - \cos^3 x}{\tan^2 x}$

(iii) $\operatorname*{Lt}_{x \to 0} \dfrac{x^2 \sin \frac{1}{x}}{\sin x}$

(iv) $\operatorname*{Lt}_{x \to \infty} \{x - \sqrt{(x - a)(x - b)}\}$

(b) Let $A \subseteq R$, let f and g be functions on A to R. Suppose that $c \in A$ and that f and g are continuous at c, then show that $f + g$, $f - g$ and $f . g$ are continuous at c. 5

UNIT—II

3. *(a)* Discuss the continuity of the following functions at points indicated : 2+4=6

(i) $f(x) \quad \begin{cases} \dfrac{x^4 \quad 4x^3 \quad 2x}{\sin x}, & if \ x \quad 0 \\ 0, & if \ x \quad 0 \end{cases}$

at the point $x \quad 0$.

(ii) $f(x) \quad |x| \quad |x \quad 1| \quad |x \quad 2|$

at the point $x \quad 1$ and $x \quad 2$.

(b) If the function $f : R \quad R$ is differentiable at the point $c \quad R$, then show that the function f will be continuous at c. 4

(c) If (x) and (x) are differentiable functions, then by using the definition of derivative, show that

$$\frac{d}{dx}\{ (x) \quad (x)\} \quad (x) \frac{d \ (x)}{dx} \quad (x) \frac{d \ (x)}{dx}$$ 4

4. *(a)* State and prove mean value theorem. 6

(b) By using the definition of derivative, show that

$$\frac{d}{dx}(\cot x) \quad \operatorname{cosec}^2 x$$ 4

(c) Show that the function $f(x)$ defined by

$$f(x) \quad \begin{cases} 3 \quad 2x, & \text{for } \frac{3}{2} \quad x \quad 0 \\ 3 \quad 2x, & \text{for } 0 \quad x \quad 3/2 \end{cases}$$

is continuous but not differentiable at $x \quad 0$. 4

UNIT—III

5. *(a)* Show that

$$\frac{1}{1 \quad x} \quad \frac{2x}{1 \quad x^2} \quad \frac{4x^3}{1 \quad x^4} \quad \frac{8x^7}{1 \quad x^8} \quad \cdots \quad \frac{1}{1 \quad x}$$

where $0 \quad x \quad 1$. 5

(b) If $y \quad \cos(m \sin^{\ 1} x)$, then show that

(i) $(1 \quad x^2)y_2 \quad xy_1 \quad m^2 y \quad 0$

(ii) $(1 \quad x^2)y_{n \ 2} \quad (2n \quad 1)xy_{n \ 1}$

$(m^2 \quad n^2)y \quad 0$

2+3=5

(c) State and prove Leibniz's theorem of successive differentiation. 4

6. *(a)* Find the extreme values of the following functions if they exist : 3+3=6

(i) $f(x) \quad 2x^3 \quad 21x^2 \quad 36x \quad 20$

(ii) $f(x) \quad x^3 \quad 9x^2 \quad 24x \quad 12$

(b) By using L'Hospital's rule, find the following limits : 4+4=8

(i) $\displaystyle \operatorname*{Lt}_{x \to 1} \left(\frac{x}{x-1} - \frac{1}{\log x} \right)$

(ii) $\displaystyle \operatorname*{Lt}_{x \to 0} \left(\frac{\tan x}{x} \right)^{1/x}$

UNIT—IV

7. *(a)* Show that a sequence of real numbers is convergent if and only if it is a Cauchy sequence. 10

(b) Define convergent sequence. Show that a convergent sequence in *R*, must converges to a unique point. 1+3=4

8. *(a)* Write the root test and ratio test for convergence of a series. 2+2=4

(b) Examine the convergence of the following series : 5+5=10

(i) $\displaystyle \frac{1^2}{2} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + \frac{4^2}{2^4} + \frac{5^2}{2^5} + \cdots$

(ii) $\displaystyle \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots$

UNIT—V

9. *(a)* Define partition of an interval, tagged partition of an interval and Riemann sum of a function. 1+2+2=5

(b) State and prove fundamental theorem of calculus. 6

(c) Suppose that f and g are Riemann integrable function on $[a, b]$ and if $f(x) \le g(x)$, $\forall x \in [a, b]$, then show that

$$\int_a^b f(x)dx \le \int_a^b g(x)dx$$ 3

10. *(a)* Find the value of

$$\oint_C \{(x + y^2)dx + (x^2 - y)dy\}$$

taken in the clockwise sense along the closed curve C, formed by $y^3 = x^2$ and the chord joining (0, 0) and (1, 1). 5

(b) Compute the integral $\int_C xy\,dx$, along the arc of the parabola $x = y^2$ from (1, −1) to (1, 1). 4

(c) Evaluate

$$f(x, y)dxdy$$
$$R$$

over the rectangle R [0, 1; 0, 1]

where

$$f(x, y) \quad \begin{array}{ll} x \quad y, & \text{if } x^2 \quad y \quad 2x^2 \\ 0, & \text{otherwise} \end{array}$$ 5

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