M.Tech Odd Semester (CBCS) Exam., December—2017

COMPUTER SCIENCE AND ENGINEERING

(1st Semester)

Course No. : MCSECF–01

(Mathematical Foundation for Computer Science)

Full Marks : 50
Pass Marks : 15

Time : 2 hours

Note :
1. Answer any **five** questions.
2. Begin each answer in a new page.
3. Answer parts of a question at a place.
4. Assume reasonable data wherever required.
5. The figures in the margin indicate full marks for the questions.

1. *(a)* We write $A \sim B$, if there is a bijection between A and B. Show that ~ is an equivalence relation. 5

(b) Show that G is a group such that for all $a, b \quad G \; (ab)^2 \quad a^2 b^2$, G is an Abelian group. 5

2. *(a)* If $A \quad \{a, b, c\}$, $B \quad \{1, 2, 3, 4, 5, 6\}$ and $f \quad \{(a, 1), (b, 2), (c, 3)\}$, then prove that f is a bijective function. 5

(b) Reduce the Boolean expression

$$A[B \quad C \;\; (AB \quad AC\;)\;]$$ 5

3. *(a)* Prove validity of the following sequence : 5

$$p \quad q, r \quad s \vdash p \quad r \quad q \quad s$$

(b) Prove or disprove the validity of the following arguments : 5

"All clear explanations are satisfactory. Some excuses are unsatisfactory. Hence, some excuses are not clear explanations."

4. *(a)* Find the complexity of $T(n) \quad 2^n \; T \; \frac{n}{2} \quad n^n$. 4

(b) Find the complexity of the function shown below : 6

```
void function (int n)
{
 if (n    1)
  return;
 if (n   1)
 {
  printf (" * ");
  function (n/2);
  function (n/2);
 }
}
```

5. *(a)* For 5 5 matrix, what is the time complexity of straight forward iterative matrix multiplication and Strassen's algorithm. Which one is better for the above matrix? 2+2+1=5

(b) Discuss merge sort and its complexity. 5

6. *(a)* Prove that a given connected graph G is an Euler graph if and only if all vertices of G are of even degree. 5

(b) Prove that a graph with at least one edge is 2-chromatic if and only if it has no circuit of odd length. 5

7. *(a)* Design an NFA for the language L = all strings over {0, 1} that have at least two consecutive 0's and 1's. 5

(b) Write REs for the following language over the alphabet $\{a, b, c\}$: 5

(i) All strings with an even number of symbols (consider '0' to be an even number)

(ii) All strings with an even number of symbols with the second symbol being b

8. *(a)* Write a context-free grammar (CFG) for the language $L(G) \quad \{ww^R : w \quad \{0,1\}^*\}$. 5

(b) Design a push-down automata (PDA) for the following language : 5

$$L \quad \{a^n b^n : n \quad 0\}$$

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